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Geometric Approaches to Bus Scheduling in Northern Ontario

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Abstract

Aims: To study the current bus network of Ontario Northland from geometric point of view. Identify the best locations for the bus depots, considering different scenarios and number of depots. Provide insight into the best ways to open a new depot, relocate one or more of the existing depots or close a depot. Provide alternative schedule and compare it to the current schedule being used by Ontario Northland Passenger Division.

Study design: We develop a mathematical model of the bus network using the discrete k-center and k-median formalisms and study the model numerically.

Place and Duration of Study: Department of Computer Science and Mathematics, Nipissing University, between April and August 2012.

Methodology: We study the road network, represented by 11 main locations using the discrete k-center and discrete k-median approach for $k = 1, 2, 3$. The distances in the model are the actual road distances along the bus network, rather than distances on the map. We develop an ad-hoc algorithmic approach, given the small size of the problem and obtain numerical characteristics of the suitability of each site, pair of sites, and triple of sites as depot locations.

Results: We present the suitability and modified suitability numbers for each site as depot, for the top 25 pairs of sites, and for the top 25 triples of sites. Further, we present the top pairs and triples under the assumption that one of the current depots in North Bay and Sudbury is kept and the other is moved, or that a third depot is added to the two existing ones. Based on these results, we present a sample schedule, based on depots in North Bay and Matheson - the best pair of depots according to our analysis. We compare it to the current schedule used by ONTC and note that it

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realizes substantial time savings over the longer routes, e.g. over 2 hours saving on a 13-hour route between Toronto and Timmins. It has to be noted that all connection times are either the same or better, given that the proposed schedule has the same number of trips as the current one.
Conclusion: The current location of the two bus depots operated by Ontario Northland in North Bay and Sudbury is non-optimal. The optimal location for two or three bus depots necessarily includes a northern location, farther north from the current two. Just based on the relocation of the depots or by opening a third bus depot north of the existing two, substantial savings in travel time can be obtained, without increasing the overall number of trips and thus the resources needed. We suggest that the research can be extended to include inventory and workforce considerations. A software tool that can be integrated into Ontario Northland's enterprise operation system should be written, incorporating the results of this study as well as those possible additions/extensions suggested above.

Keywords: k-median; k-center; optimal scheduling; facility location; transportation; operations research
2010 Mathematics Subject Classification: 90B06; 90B35; 90B80; 90C47

1 Introduction

This research was initiated by a case study presented at the 2^{nd} Annual Workshop on Algorithmic Graph Theory held in North Bay on May 16–20, 2011 [1]. This report summarizes the findings/suggestions of the academic panel. It extends them in the direction of a mathematical model that addresses the bus scheduling problem for Ontario Northland.

Ontario Northland is a vital provider of transportation of both people and goods throughout Northern Ontario. Established in 1902 and with over 1,100km of track and passenger transport by rail or bus to over 55 communities in Ontario, it is quite a large and complex operation [2]. As of now, Ontario Northland has its two main bus depots located in Sudbury and North Bay with its actual headquarters in North Bay. This paper will explore other possible locations for these depots and their suitability based on distance using a discrete k-center and k-median approach. For simplicity's sake, only specific stops along Ontario Northland routes will be considered as possible depots such as those which are points of connection and those which Ontario Northland has deemed as its main stops.

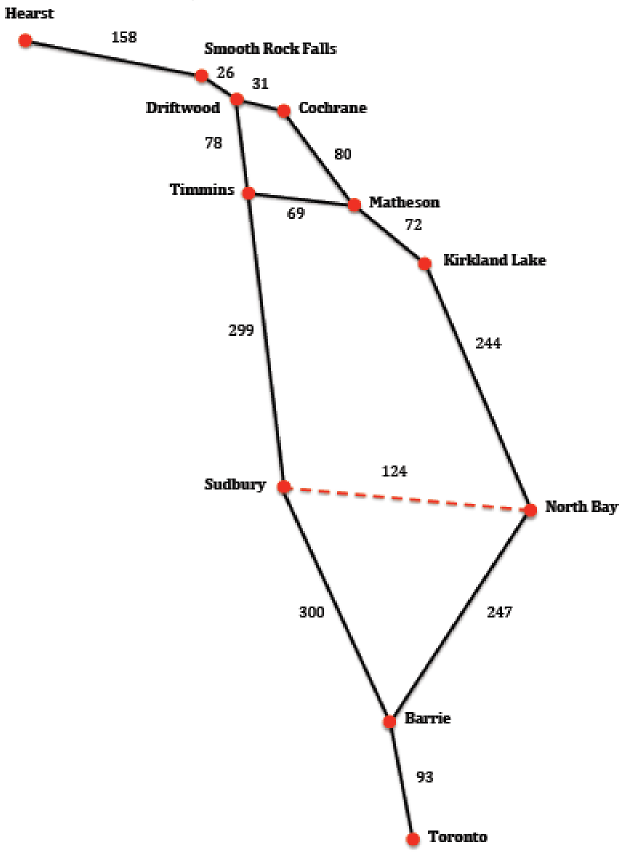
In the end, 11 possible depot sites will be considered in this paper. These main depot sites are: Toronto, Barrie, Sudbury, North Bay, Kirkland Lake, Timmins, Matheson, Cochrane, Driftwood, Smooth Rock Falls and Hearst.

2 The k-Center Problem

2.1 The Method

To solve the discrete k-center/k-median problem is not trivial. Significant amount of research in the last two decades has been devoted to them. For example, in the case of the discrete 2-center problem, there best algorithm is proposed by Agarwal, Sharir and Welzl [3] in 1997. Its running time is $O(n^{4/3}\log^5 n)$, where n is the number of sites in the network. However, in general these algorithms are very complex and, given that there are only 55 possible pairs of depots and 165 possible triples in this specific application, it makes more sense to use an ad-hoc approach.

Figure 1: The road network served by Ontario Northland



2.2 The Execution

Each possible pair and then triple of depots will be considered one at a time. For example, in the case that we are restricted to 2 depots, given depots a and b, depot a will be assigned all towns/cities to be considered that are closer to it than to depot b. Similarly, depot b will be assigned all towns/cities which are closer to it than to depot a. (Obviously, a depot will always be assigned at least one stop, namely itself since clearly it is closer to itself than any other possible depot). Then, the maximum distances between each depot and its furthest stop assigned to it will be determined. These two maximums then become the respective radii of the circles centered at depot a and depot b which encompass all the necessary stops assigned to each depot. The maximum of these radii will become the number assigned to that particular pairing of depots. We call this number the *suitability number* for that pair. The goal is to find the pair (and then the triple) of depots which has the least suitability number and is thus the most suitable pair (or triple) according to the model.

2.3 The Data

Table 1 is a table of the distances from each of the stops being considered to all the other stops. This table of distances contains the data which is the basis for the following investigations.

Table 1: Distances Between Towns/Cities to be Considered as Potential Depot Sites

Location	Tor	Bar	Sudb	N Bay	K Lake	Timm	Math	Drif	Coch	Sm R Falls	Hearst
Toronto	0	93	393	340	584	692	656	770	736	793	951
Barrie	93	0	300	247	491	599	563	677	643	700	858
Sudbury	393	300	0	124	368	299	368	377	448	403	561
North Bay	340	247	124	0	244	385	316	427	396	453	611
Kirkland Lake	584	491	368	244	0	141	72	183	152	209	367
Timmins	692	599	299	385	141	0	69	78	109	104	262
Matheson	656	563	368	316	72	69	0	111	80	137	295
Driftwood	770	677	377	427	183	78	111	0	31	26	184
Cochrane	736	643	448	396	152	109	80	31	0	57	215
Smooth Rock Falls	793	700	403	453	209	104	137	26	57	0	158
Hearst	951	858	561	611	367	262	295	184	215	158	0

3 One-Center Problem

The investigation begins with a brief examination into the optimal position for a depot if there were to be only one. Of course, the optimality of this location for a depot will depend solely on the distances from that location to the other locations as listed in Table 1. Intuitively, the obvious optimal position as far as travelling distances are concerned would be the station which is center-most in the overall map of road distances. For instance, clearly Hearst and Toronto, as the extreme points, would not be intelligent choices for a single depot if the distance between said depot and all other stations was to be minimized. Let us consider one of the more centrally located stations such as Sudbury. We simply determine the station which is furthest from Sudbury, which is Hearst at 561km, and the distance from Sudbury to Hearst becomes the maximum distance to be travelled if Sudbury were the sole depot; thus, the suitability number for Sudbury is 561. It can be quickly deduced from Table 1 that Sudbury is in fact the best choice for such a depot, although North Bay, with a maximum distance to be travelled of 611km would not be an unrealistic choice either. (See Table 2 for a complete set of the results for the 1-Center Problem)

Table 2: One-Center Data

Depot	Maximum Distance Travelled
Sudbury	561
Kirkland Lake	584
North Bay	611
Matheson	656
TImmins	692
Cochrane	736
Driftwood	770
Smooth Rock Falls	793
Barrie	858
Toronto	951
Hearst	951

4 Two-Center Problem

A more interesting question is what pairing of stations is optimal given the road distance configuration. To determine the optimal pairing, each of the 55 possible pairs must be considered. Given a pair a and b, we must then assign all other stations to either a or b depending on which depot location is closer to each station. Then, the maximum of the distances to be travelled for each depot is found and the maximum of these two measures will serve as the measure of the pairing's suitability.

For example, given the pairing Barrie and Kirkland Lake, consider Table 3 of the distances from Barrie and Kirkland Lake to each other station. Toronto will be assigned to Barrie as the distance from Barrie to Toronto as given by Table 3 is 93km whereas the distance from Kirkland Lake to Toronto is over 500km. Continuing in this fashion we get the following set of results shown in Tables 4 and 5. Toronto, Barrie and Sudbury are assigned to the Barrie depot in this pairing and the other locations are assigned to Kirkland Lake. Considering the distances in Tables 4 and 5 we see that the maximum distance to be travelled from the Barrie depot would be 300km while the maximum distance from the Kirkland Lake depot to be travelled would be 367km. Therefore, as mentioned above, the suitability number for this pairing is the maximum of these two numbers. (In this example it would be 367km).

Table 3: Example: Two-Center Data for Depots at Barrie and Kirkland Lake - Distances - Cities Assigned to Barrie

Location	Barrie	Kirkland Lake
Toronto	93	584
Barrie	0	491
Sudbury	300	368
North Bay	247	244
Kirkland Lake	491	0
Timmins	599	141
Matheson	563	72
Driftwood	677	183
Cochrane	643	152
Smooth Rock Falls	700	209
Hearst	858	367

Table 4: Example: Two-Center Data for Depots at Barrie and Kirkland Lake - Cities Assigned to Barrie

Location	Barrie
Toronto	93
Barrie	0
Sudbury	300

Table 5: Example: Two-Center Data for Depots at Barrie and Kirkland Lake - Cities Assigned to Kirkland Lake

Location	Kirkland Lake
North Bay	244
Kirkland Lake	0
Timmins	141
Matheson	72
Driftwood	183
Cochrane	152
Smooth Rock Falls	209
Hearst	367

Continuing in this fashion, a suitability number for each of the possible pairings was determined. The top 25 pairings and their suitability numbers are displayed in Table 6 organized from most to least suitable.

Table 6: Two-Center Results - Top 25 Ranked Most to Least Suitable According to Suitability Number

Rank	Pair	Suitability Number
1	North Bay, Matheson	295
2	Barrie, Timmins	300

2	Barrie, Matheson	300
2	Barrie, Driftwood	300
2	Barrie, Cochrane	300
2	Barrie, Smooth Rock Falls	300
3	Toronto, Timmins	340
3	North Bay, Timmins	340
3	North Bay, Driftwood	340
3	North Bay, Cochrane	340
3	North Bay, Smooth Rock Falls	340
3	North Bay, Hearst	340
4	Barrie, Kirkland Lake	367
4	Barrie, Hearst	367
4	North Bay, Kirkland Lake	367
5	Toronto, Kirkland Lake	368
5	Toronto, Matheson	368
6	Toronto, Driftwood	377
7	Toronto, Cochrane	393
7	Toronto, Smooth Rock Falls	393
7	Toronto, Hearst	393
7	Sudbury, Kirkland Lake	393
7	Sudbury, Timmins	393
7	Sudbury, Matheson	393
7	Sudbury, Driftwood	393

Although North Bay and Sudbury are among many of the 25 most suitable pairings (in fact, North Bay and Matheson is the optimal pairing), the pairing of North Bay and Sudbury together, which is the current situation, is not among these 25 most suitable. Assigning the depots to North Bay and Sudbury yields a suitability number of 561 which is almost twice that of the optimal pairing and has a ranking of 8 in the overall ranking of the suitability of the pairs. Thus, if ONTC were to move their depot in Sudbury to Matheson this would almost halve the maximum distance a bus travelled one way to any destination from a depot.

4.1 Modified Suitability Number Based on Distance Between Depots

This result, while interesting, does not take into account the distance between the two depot locations. Such a consideration is very important to the overall practicality and efficiency of the bus route system. For example, if the two depots were located a large distance from one another, if one depot were to require an extra bus or anything of this nature from the other depot, logistically the locations would not be ideal. However, requiring the depots to be very close together is unwise as well because this would likely increase the radius of their service areas and consequently the suitability number for such a pairing as well. In order to take into consideration the distance between the two depot locations in each pairing we will simply add the distance between the two locations to the suitability number to produce a modified suitability number. The top 25 results for these modified suitability numbers can be found in Table 7 organized from most to least suitable.

Table 7: Two-Center Results: Top 25 Ranked Most to Least Suitable According to Modified Suitability Number

Rank	Pair	Suitability Number	Modified Suitability Number
1	North Bay, Matheson	295	610
2	North Bay, Kirkland Lake	367	611
3	Kirkland Lake, Matheson	584	656
4	Sudbury, North Bay	561	685
5	Sudbury, Timmins	393	692
6	North Bay, Timmins	340	725
6	Kirkland Lake, Timmins	584	725
6	Timmins, Matheson	656	725
7	North Bay, Cochrane	340	736
7	Kirkland Lake, Cochrane	584	736
7	Matheson, Cochrane	656	736
8	Sudbury, Kirkland Lake	393	761
8	Sudbury, Matheson	393	761
9	North Bay, Driftwood	340	767
9	Kirkland Lake, Driftwood	584	767
9	Matheson, Driftwood	656	767
9	Driftwood, Cochrane	736	767
10	Sudbury, Driftwood	393	770
10	Timmins, Driftwood	692	770
11	North Bay, Smooth Rock Falls	340	793
11	Kirkland Lake, Smooth Rock Falls	584	793
11	Matheson, Smooth Rock Falls	656	793
11	Cochrane, Smooth Rock Falls	736	793
12	Sudbury, Smooth Rock Falls	393	796
12	Timmins, Smooth Rock Falls	692	796

Interestingly, the pair North Bay and Matheson is once again the optimal choice; however, with the modified suitability number, the current situation of North Bay and Sudbury being the two depots ranks 4^{th} as opposed to 8^{th} with the suitability number alone. Also, considering that North Bay and Sudbury are both large cities in Northern Ontario, especially when compared to other members of higher ranking pairs such as Matheson and Kirkland Lake, the pair North Bay, Sudbury is a logistically sound choice in terms of minimizing both the distance travelled by any one bus and also minimizing the distance and thus the cost of transporting goods and people from one depot to another provided the direct route North Bay-Sudbury is available to ONTC; however, presently it is not.

5 Three-Center Problem

There are 165 possible triples and three main questions to ask when considering the 3-center problem: assuming that we cannot move the two depots currently located at Sudbury and North Bay, what would be the most logistically efficient location for a third depot? What if we can move only one of the existing depots? What would be the more efficient choices for a second and third depot? And lastly, if one could relocate any or all depots, what would the optimal locations for these depots be?

5.1 Adding a Third Depot to the Existing Situation

If ONTC did not wish to move their current depots from Sudbury or North Bay, we must consider all the triples which include both these depots. Displayed in Table 8 is an ordered list of the triples which contain both Sudbury and North Bay. Clearly, the third depot should be located at one of the following locations: Timmins, Matheson, Driftwood, Cochrane, Smooth Rock Falls or Hearst. Given this list, Timmins and Cochrane seem the more desirable options given that Timmins is a rather large city compared to the others in the list (some of which are very small towns) and Cochrane is one end of the Polar Bear Express train which is the only ground connection to Moosonnee, ON from any of its southern neighbours. Moosonnee,ON can only be reached by plane or train and thus, the Polar Bear Express, as the only train which runs to Moosonnee, plays a vital role in the survival of the community there.

However, if we begin taking into account the distances between each of the depots and generate a modified suitability number as was done in the 2-center problem, we may be able to avoid the situation where there are multiple options for a third depot which are all equally suitable in terms of the numbers alone. To calculate the Modified Suitability Number for three depots a, b, and c, we will add the distance between each of the depots (i.e. d(a,b), d(a,c), d(b,c)) to the suitability number.

Taking into account the modified suitability numbers, Matheson and Timmins are equally suitable choices for the third depot; however, considering that North Bay, Matheson is the optimal pair in the 2-center results with both the suitability number and the modified suitability number, Matheson is the more desirable choice for this third depot.

Table 8: Three-Center Data: Adding a Third Depot to the Existing Situation - Ranked Most to Least Suitable by Suitability Number

Rank	Triple	Suitability Number	Modified Suitability Number
1	Sudbury, North Bay, Timmins	340	1148
1	Sudbury, North Bay, Matheson	340	1148
1	Sudbury, North Bay, Driftwood	340	1268
1	Sudbury, North Bay, Cochrane	340	1308
1	Sudbury, North Bay, Smooth Rock Falls	340	1320
1	Sudbury, North Bay, Hearst	340	1636
2	Sudbury, North Bay, Kirkland Lake	367	1103
3	Toronto, Sudbury, North Bay	561	1418
3	Barrie, Sudbury, North Bay	561	1232

5.2 Keeping the North Bay Depot

We must now ask ourselves what our choices would be if ONTC was willing to move one of the depots from Sudbury or from North Bay if this would improve the suitability (or modified suitability) of the overall operation. Firstly, let us explore what would happen to the suitability numbers if we kept North Bay (as the current headquarters of ONTC) and allowed the depot at Sudbury to be moved to another location. If we simply wanted to minimize the maximum distance that buses from each depot must travel (a.k.a. the suitability numbers), the tie for the optimal choice would be between Toronto, North Bay, Driftwood and Barrie, North Bay, Driftwood with suitability number 184. This means that with either of these two triples, buses leaving any of the depots would travel a maximum of 184km one way. Since the routes would be short, the buses could run more often and the drivers would probably not need to be accommodated overnight anywhere which could happen if they were required to drive more than the maximum amount of kilometres allowed in one 24 hour period. In fact, the top 10 results displayed in Table 9 suggest that the depot at Sudbury should be moved to a northern location such as Matheson, Driftwood, Cochrane, Smooth Rock Falls or Hearst while the third depot can be located at either Toronto or Barrie without affecting the suitability number for that triple.

If the distance between the three depots is taken into account and the modified suitability numbers are calculated, the triple Barrie, North Bay, Driftwood has a modified suitability number of 1,535km while the triple Toronto, North Bay, Driftwood clearly has a higher modified suitability number because Toronto is further from North Bay and Driftwood as it is South of Barrie and the bus would have to travel through Barrie and onward to reach Toronto. However, if we only consider the modified suitability numbers, the optimal choice is easily North Bay, Kirkland Lake, Matheson with a modified suitability number of 972km - much less than the triples considered with only the suitability number as optimal. The suitability number for this triple is 340km. Recall that if we require Sudbury and North Bay to be two of the three depots, the optimal triples have a suitability number of 340km. Thus, this number is not unreasonably high, although a maximum distance travelled one way for a bus of under 200km does sound more appealing.

There are more factors that would have to be considered in making these choices such as the volume of passenger traffic from these depots. For example, if very few people travel further north than Driftwood, perhaps it would not be beneficial to have such a short distance to be travelled by each coach because the passenger traffic may not be able to support many successive runs of the same route in one day. On the other hand, if there are areas through which the passenger traffic is consistently high, even too much for one coach at times, having those routes run more times in one day could alleviate the need for two buses to run the same route at the same time to accommodate high passenger numbers. Also, running coaches more times in a day would clearly make travel by coach more convenient for people who need more options for departure and arrival times than are currently offered by ONTC.

Table 9: Three-Center Results: Keeping a Depot at North Bay - Top 20
Ranked Most to Least Suitable According to Suitability Number

Rank	Triple	Suitability Number	Modified Suitability Number
1	Toronto, North Bay, Driftwood	184	1721
1	Barrie, North Bay, Driftwood	184	1535
2	Toronto, North Bay, Smooth Rock Falls	209	1795
2	Barrie, North Bay, Smooth Rock Falls	209	1609
3	Toronto, North Bay, Cochrane	215	1687
3	Barrie, North Bay, Cochrane	215	1501
4	Toronto, North Bay, Timmins	262	1679
4	Barrie, North Bay, Timmins	262	1493
5	Toronto, North Bay, Matheson	295	1607
5	Toronto, North Bay, Hearst	295	2197
5	Barrie, North Bay, Matheson	295	1421
5	Barrie, North Bay, Hearst	295	2011
6	North Bay, Kirkland Lake, Timmins	340	1110
6	North Bay, Kirkland Lake, Matheson	340	972
6	North Bay, Kirkland Lake, Driftwood	340	1194
6	North Bay, Kirkland Lake, Cochrane	340	1132

6	North Bay, Kirkland Lake, Smooth Rock Falls	340	1246
6	North Bay, Kirkland Lake, Hearst	340	1562
6	North Bay, Timmins, Matheson	340	1110
6	North Bay Timmins, Driftwood	340	1230

5.3 Keeping the Sudbury Depot

If Sudbury is to remain a depot while the other two locations can be modified, the top 10 choices for these triples mirror those in the North Bay case as shown in Table 10. The suitability numbers are the same with Sudbury being substituted for North Bay. Thus, again we have Sudbury with a northern location such as Kirkland Lake, Timmins, Matheson, Driftwood, Cochrane, Smooth Rock Falls or Hearst and either Toronto or Barrie. Sudbury, Kirkland Lake and either Timmins or Matheson yield the lowest modified suitability number while having a suitability number over twice the optimal achieved with Toronto, Sudbury, Driftwood or Barrie, Sudbury, Driftwood. Again, other parameters must be taken into account such as passenger traffic levels in order to make these decisions.

Table 10: Three-Center Results: Keeping a Depot at Sudbury - Top 20 Ranked Most to Least Suitable According to Suitability Number

Rank	Triple	Suitability Number	Modified Suitability Number
1	Toronto, Sudbury, Driftwood	184	1724
1	Barrie, Sudbury, Driftwood	184	1538
2	Toronto, Sudbury, Smooth Rock Falls	209	1798
2	Barrie, Sudbury, Smooth Rock Falls	209	1612
3	Toronto, Sudbury, Cochrane	215	1792
3	Barrie, Sudbury, Cochrane	215	1606
4	Toronto, Sudbury, Timmins	262	1646
4	Barrie, Sudbury, Timmins	262	1460
5	Toronto, Sudbury, Matheson	295	1712
5	Barrie, Sudbury, Matheson	295	1526
6	Toronto, Sudbury, Kirkland Lake	367	1721
6	Toronto, Sudbury, Hearst	367	2272
6	Barrie, Sudbury, Kirkland Lake	367	1526
6	Barrie, Sudbury, Hearst	367	2086
7	Sudbury, Kirkland Lake, Timmins	393	1201
7	Sudbury, Kirkland Lake, Matheson	393	1201
7	Sudbury, Kirkland Lake, Driftwood	393	1321
7	Sudbury, Kirkland Lake, Cochrane	393	1361
7	Sudbury, Kirkland Lake, Smooth Rock Falls	393	1373
7	Sudbury, Kirkland Lake, Hearst	393	1689

5.4 Without Restrictions on Depot Locations

Given the option to place the three depots at any of the locations without restriction, it becomes clear that the modified suitability number is skewed in favour of triple which are very close together. Because of this skewness of the results, the top 25 triples in Table 11 are all made up of locations which are very close together. (Note that the triples in Table 11 are sorted by their modified suitability number).Take the optimal triple, Timmins, Driftwood and Cochrane, clearly its suitability number is very high at 692km. Despite this high suitability number, the extreme closeness of these three locations to one another means that the modified suitability number is only a little more than 100km larger than suitability number so overall the modified suitability numbers ends up being very low. Thus, it is important to consider both the suitability number and the modified suitability number to get a more accurate result.

Considering the results as shown in Table 11, North Bay, Kirkland Lake, Matheson is the first triple to appear in the table with a reasonably low suitability number and a modified suitability number of under 1,000km. Another relatively well suited triple is Sudbury, North Bay, Kirkland Lake with a suitability number of 367km and a modified suitability number of just over 1,100km. Considering the obvious enormous cost both in finances and time, to relocate and/or establish a new depot, because the benefit of moving depots from North Bay and/or Sudbury, does not seem to be very high, the best option given the current situation would probably be to simply add a third depot at Kirkland Lake while keeping the current depots at North Bay and Sudbury as they are. Although, it would be possible to dramatically shorten the maximum distance travelled from one depot to under 200km, the triples which allow this also have relatively high modified suitability numbers and would require the relocation of one of the existing depots.

Table 11: Three-Center Results: Without Restrictions on Depot Locations - Top 25 Ranked Most to Least Suitable According to Modified Suitability Number

Rank	Triple	Suitability Number	Modified Suitability Number
1	Timmins, Driftwood, Cochrane	692	801
2	Driftwood, Cochrane, Smooth Rock Falls	736	850
3	Kirkland Lake, Timmins, Matheson	584	866
4	Matheson, Driftwood, Cochrane	656	878
5	Kirkland Lake, Matheson, Cochrane	584	888
6	Timmins, Driftwood, Smooth Rock Falls	692	900
7	Timmins, Matheson, Driftwood	656	914
7	Timmins, Matheson, Cochrane	656	914
8	Matheson, Driftwood, Smooth Rock Falls	656	930
8	Matheson, Cochrane, Smooth Rock Falls	656	930
9	Kirkland Lake, Matheson, Driftwood	584	950
9	Kirkland Lake, Driftwood, Cocrhane	584	950
10	Timmins, Cochrane, Smooth Rock Falls	692	962
11	Timmins, Matheson, Smooth Rock Falls	656	966
12	North Bay, Kirkland Lake, Matheson	340	972
13	Kirkland Lake, Timmins, Driftwood	584	986
13	Kirkland Lake, Timmins, Cochrane	584	986
14	Kirkland Lake, Matheson, Smooth Rock Falls	584	999
15	Kirkland Lake, Driftwood, Smooth Rock Falls	584	1002
15	Kirkland Lake, Cochrane, Smooth Rock Falls	584	1002
16	Kirkland Lake, Timmins, Smooth Rock Falls	584	1038
17	Sudbury, North Bay, Kirkland Lake	367	1103
18	North Bay, Kirkland Lake, Timmins	340	1110
18	North Bay, Timmins, Matheson	340	1110
19	Sudbury, Timmins, Matheson	393	1129

6 The k-Median

Thus far in the paper, the optimal positions for one, two and three depots were considered in terms of the distances from each depot to the towns/cities it would service. The main goal with this approach was to minimize the maximum distance travelled from each depot to the furthest town/city that it services. With the k-center approach we are able to establish systems with shorter routes and that have more locations serviced separately. In this way, we can establish more flexible schedules with shorter wait times for passengers transferring at depots and to better accommodate drivers minimum and maximum hours requirements.

Now we will consider the sum of the distances from each depot to each town/city that it services to determine a measure of the median; thus, the k-median model corresponds to the sum of all distances if every location is serviced by a route from its closest depot with respect to the k-center model. Using the k-median model we will be able to determine the combinations of depot locations that would have the shortest possible overall route length and hence the least cost in gas and vehicle mileage. Also, we can determine routes which would serve multiple locations more efficiently using the k-median model.

6.1 The Method

Having already compiled the data for the k-center model, computing the data for the 1-median, 2-median and 3-median models is very easily done. We simply assign all towns/cities to their nearest depot as in the k-center model and then take the sum of the distances from these locations to there assigned depot and that number will be the median for that scenario.

For example, in a 2-median model with depots located at Barrie and Kirkland Lake (to use the same example as in the 2-center model) Toronto and Sudbury are closer to Barrie than they are to Kirkland Lake so they will be assigned to the depot at Barrie while all other towns/cities being considered will be assigned to Kirkland Lake. Displayed in Tables 12 and 13 are the distances from each of the depots to the towns/cities assigned to them and the sum of these distances for each depot is displayed in the last row of the respective table.

Table 12: Example: Two-Median Data for Depots at Barrie and Kirkland Lake - Cities Assigned to Barrie

Location	Barrie
Toronto	93
Barrie	0
Sudbury	300
Total Distance	393

Table 13: Example: Two-Median Data for Depots at Barrie and Kirkland Lake - Cities Assigned to Kirkland Lake

Location	Kirkland Lake
North Bay	244
Kirkland Lake	0
Timmins	141
Matheson	72
Driftwood	183
Cochrane	152
Smooth Rock Falls	209
Hearst	367
Total Distance	1368

Therefore, the total distance to be travelled from each despot to its assigned towns/cities in this example would be the sum of the individual distance totals which in this example is 393 + 1368 = 1761. Thus, the median measurement for the pair of depots Barrie and Kirkland Lake would be 1761.

7 One-Median Scenario

To calculate the median for each possible depot location in a 1-center model we need only find the sum of the distances in each row or column of Table 1. The results of this operation are displayed in Table 14.

Table 14: One-Median Results - Ranked According to Median from Shortest to Longest Distance

Location	Median
Matheson	2667
Timmins	2738
Kirkland Lake	2811
Cochrane	2827
Driftwood	2864
Smooth Rock Falls	3040
North Bay	3543
Sudbury	3641
Hearst	4462
Barrie	5171
Toronto	6008

Clearly the results for the 1-median scenario differ greatly from those of the 1-center problem. The optimal location for a depot in the 1-center problem was Sudbury however if we consider the medians we find that Sudbury is not even among the upper half of the results. Also, Matheson, which has the lowest median, is 4^{th} when using the 1-center approach. This means that while Sudbury may have the least maximum distance from it to any other location considered, it may not have the shortest possible overall route length or provide a very efficient system for servicing the other locations. To demonstrate this conclusively, one would have to consider the distribution of these towns/cities along the road network and to consider all possible routes which may service multiple locations thus developing a system of routes. Such considerations are not within the scope of this paper; we simply seek to demonstrate the potential of such knowledge by providing useful insights while only scraping the surface. However, we will demonstrate an example, later in the paper, of how the geometric considerations can help us obtain a better schedule even when computed by hand.

8 Two-Median Scenario

In considering the 2-median scenario, the process is the same: we find the sum of the distances from each of the depots to their assigned towns/cities. The top 20 results for the two-median approach are displayed in Table 15. The pair with the least median is North Bay, Smooth Rock Falls. In the two-center approach this pair is third in the ranking of the suitability and 11^{th} in the ranking of the modified suitability. Considering that there are 55 possible pairs, 11^{th} is not necessarily a poor ranking in the overall situation given the suitability ranking and the median. The pair North Bay, Smooth Rock Falls may be a good candidate for both minimizing the maximum distance travelled and for minimizing the overall length of the bus routes to be run from these depots.

There are other pairings to consider as well. The pairs of depots Barrie-Driftwood and Barrie-Cochrane have an even lower suitability number than the pair North Bay-Smooth Rock Falls; however, these two pairs are not even among the top 25 results when the modified suitability number is considered. This means that while the maximum distance from a depot to one of its locations is minimized and the overall cost of running buses from these depots in gas and vehicle mileage may be minimal as well, unfortunately, these two depot locations are quite far from each other relative to our network; thus, transferring buses, parts, passengers etc. from one depot to another would not be efficient.

Table 15: Two-Median Results - Top 20 Ranked According to Median from Shortest to Longest Distance

Rank	Pair	Median
1	North Bay, Smooth Rock Falls	1244
2	Barrie, Driftwood	1253
3	Barrie, Cochrane	1284
4	North Bay, Driftwood	1324
5	Barrie, Smooth Rock Falls	1331
6	North Bay, Cochrane	1355
7	Barrie, Timmins	1403
8	Barrie, Matheson	1404
9	Toronto, Driftwood	1423
10	Sudbury, Driftwood	1430
11	Sudbury, Cochrane	1461
12	Toronto, Cochrane	1470
13	North Bay, Timmins	1474

14	North Bay, Matheson	1475
15	Toronto, Timmins	1495
16	Subdury, Smooth Rock Falls	1508
17	Toronto, Smooth Rock Falls	1517
18	Toronto, Matheson	1541
19	Sudbury, Timmins	1580
20	Subdury, Matheson	1581

8.1 The Modified Median

As in our k-center model, we now would like to take into account the distance between depots in our measure of the median. Thus, we simply must add the distance between the depots to the medians to generate the modified median (See Table 16).

Table 16: Two-Median Results - Top 20 Ranked According to Modified Median from Shortest to Longest Distance

Rank	Pair	Median	Modified Median
1	North Bay, Smooth Rock Falls	1244	1697
2	North Bay, Driftwood	1324	1751
2	North Bay, Cochrane	1355	1751
3	North Bay, Matheson	1475	1791
4	Sudbury, Driftwood	1430	1807
5	North Bay, Timmins	1474	1859
6	Sudbury, Cochrane	1461	1869
7	Sudbury, Timmins	1580	1879
8	Sudbury, Smooth Rock Falls	1508	1911
9	Barrie, Cochrane	1284	1927
10	Barrie, Driftwood	1253	1930
11	Sudbury, Matheson	1581	1949
12	Barrie, Matheson	1404	1967
13	Barrie, Timmins	1403	2002
14	Barrie, Smooth Rock Falls	1331	2031
15	North Bay, Kirkland Lake	1835	2079
16	Toronto, Timmins	1495	2187
17	Toronto, Driftwood	1423	2193
18	Toronto, Matheson	1541	2197
19	Toronto, Cochrane	1470	2206

It would seem that the pair North Bay, Smooth Rock Falls is the optimal choice when considering the modified median as well. Also interesting is the fact that the pair North Bay, Matheson ranks $3^{\rm rd}$ in the modified median ranking and $1^{\rm st}$ in the modified suitability rankings and thus is an obvious candidate for further research into its optimality. Also noteworthy is the fact that among the top 10 results for the modified median, only the tenth does not contain either North Bay or Sudbury; clearly, these two cities play important roles in the geometric consideration of the ONTC road network which is encouraging considering that the current situation has its two depots located at North Bay and Sudbury.

9 Three-Median Scenario

Now, as was done with the k-center model, we will extend our median scenario to a three depot scenario and compare these results with the rankings of the triples in the 3-center problem. Therefore, first we must consider the case where ONTC wishes to keep their current depots at North Bay and Sudbury and to add a third depot in one of the other 9 locations. The choice of a third depot location that minimizes the median is Driftwood, followed closely by other northernly towns such as Cochrane and Smooth Rock Falls.

Table 17: Three-Median Results: Sudbury and North Bay - Ranked According to Median from Shortest to Longest Distance

Rank	Triple	Median	Modified Median
1	Sudbury, North Bay, Driftwood	1200	2128
2	Sudbury, North Bay, Cochrane	1231	2199
3	Sudbury, North Bay, Smooth Rock Falls	1278	2258
4	Sudbury, North Bay, Timmins	1350	2158
5	Sudbury, North Bay, Matheson	1351	2159
6	Sudbury, North Bay, Kirkland Lake	1711	2447
7	Sudbury, North Bay Hearst	1945	3241
8	Barrie, Sudbury, North Bay	2689	3360
8	Toronto, Sudbury, North Bay	2689	3546

In the 3-center problem, Driftwood, Timmins, Cochrane, Matheson, Smooth Rock Falls and Hearst were all equally suitable choices for the third depot when considering the suitability number alone. Thus, given that the triples Sudbury, North Bay and Driftwood, Cochrane, Timmins or Matheson have relatively low suitbailiy numbers, median and modified medians, these four cities are the most logistically sound locations for the third depot based on the considerations thus far.

9.1 Keeping the North Bay Depot

If ONTC were to specify that it was not willing to relocate its depot at North Bay, then we would only have to consider those triples which contain North Bay (See Table 18). With the median, the optimal solution is one in which the depots are located further away from each other in order to minimize the sum of the distances from each depot to its assigned cities/towns. Intuitively, if the depots were located in close proximity to one another the sum of the distances from each of them to their assigned locations would be relatively higher than if the depots were more distant from each other. Thus, since North Bay is a relatively central location in terms of the layout of the road network and cities/towns being considered, it would be a logically sound choice for one of the three depots while the other two would be located closer to the extreme points of the network. This is the case in the results displayed in Table 18. The top 10 triples according to median all contain either Barrie or Toronto (one of the southernmost locations) and then a northern location such as Driftwood, Cochrane or Smooth Rock Falls.

Table 18: Three-Median Results: Keeping the North Bay Depot - Top 10 Ranked According to Median from Shortest to Longest Distance

Rank	Triple	Median
1	Toronto, North Bay, Driftwood	830
1	Barrie, North Bay, Driftwood	830
2	Toronto, North Bay, Cochrane	861

2	Barrie, North Bay, Cochrane	861
3	Toronto, North Bay, Smooth Rock Falls	908
3	Barrie, North Bay, Smooth Rock Falls	908
4	Toronto, North Bay, Timmins	980
4	Barrie, North Bay. Timmins	980
5	Toronto, North Bay, Matheson	981
5	Barrie, North Bay, Matheson	981

9.2 Keeping the Sudbury Depot

If ONTC were to specify that the depot at Sudbury could not be moved then the top 10 results are almost identical to those in the scenario where North Bay must remain a depot: Sudbury becomes the central depot and it is grouped with one depot location in the southern extreme and one in the northern. Note that Hearst being relatively further from other cities/towns compared to the other northern locations such as Driftwood, Cochrane, Smooth Rock Falls, Timmins and Matheson which are all located in a small cluster results in Hearst not being among the better results for the median. Hearst is automatically at least 158km away from any other city/town in the network because it is only connected to Smooth Rock Falls and the distance between the two is 158km.

Table 19: Three-Median Results: Keeping the Sudbury Depot - Top 10 Ranked According to Median from Shortest to Longest Distance

Rank	Triple	Median
1	Toronto, Sudbury, Driftwood	830
1	Barrie, Sudbury, Driftwood	830
2	Toronto, Sudbury, Cochrane	861
2	Barrie, Sudbury, Cochrane	861
3	Toronto, Sudbury, Smooth Rock Falls	908
3	Barrie, Sudbury, Smooth Rock Falls	908
4	Toronto, Sudbury, Timmins	980
4	Barrie, Sudbury, Timmins	980
5	Toronto, Sudbury, Matheson	981
5	Barrie, Sudbury, Matheson	981

9.3 Without Restrictions on Depot Locations

Not surprisingly, if we consider the case where there are no restrictions on depot locations, Sudbury or North Bay appear in each of the triples for at least the first 20 results. In fact, as shown in Table 20, the first 10 results consist of either Toronto or Barrie as the southern depot, Sudbury or North Bay as the centrally located depot and then one of the northern locations as the third depot.

Table 20: Three-Median Results: Without Restrictions on Depot Locations- Top 20 Ranked According to Median

Rank	Triple	Median	Modified Median
1	Toronto, Sudbury, Driftwood	830	2370
1	Toronto, North Bay, Driftwood	830	2367
1	Barrie, Sudbury, Driftwood	830	2184
1	Barrie, North Bay, Driftwood	830	2181

2	Toronto, Sudbury, Cochrane	861	2438
2	Toronto, North Bay, Cochrane	861	2333
2	Barrie, Sudbury, Cochrane	861	2252
2	Barrie, North Bay, Cochrane	861	2147
3	Toronto, Sudbury, Smooth Rock Falls	908	2497
3	Toronto, North Bay, Smooth Rock Falls	908	2494
3	Barrie, Sudbury, Smooth Rock Falls	908	2311
3	Barrie, North Bay, Smooth Rock Falls	908	2308
4	Toronto, Sudbury, Timmins	980	2364
4	Toronto, North Bay, Timmins	980	2397
4	Barrie, Sudbury, Timmins	980	2178
4	Barrie, North Bay. Timmins	980	2211
5	Toronto, Sudbury, Matheson	981	2398
5	Toronto, North Bay, Matheson	981	2293
5	Barrie, Sudbury, Matheson	981	2212
5	Barrie, North Bay, Matheson	981	2107

Thus, we must also consider the modified median which will help to ensure that while the depots are not located too close to one another, the depots do not become spread far enough from each other that travel and shipping between them becomes inefficient. Ranking the triples according to modified median yields the following results displayed in Table 21.

Table 21: Three Median-Results: Without Restrictions on Depot Locations - Top 20 Ranked According to Modified Median

Rank	Triple	Median	Modified Median
1	North Bay, Matheson, Driftwood	1093	1947
1	North Bay, Matheson, Cochrane	1155	1947
2	North Bay, Kirkland Lake, Driftwood	1102	1956
3	North Bay, Kirkland Lake, Cochrane	1195	1987
4	North Bay, Matheson, Smooth Rock Falls	1093	1999
5	Sudbury, Timmins, Driftwood	1268	2022
6	North Bay, Kirkland Lake, Smooth Rock Falls	1128	2034
7	North Bay, Kirkland Lake, Matheson	1403	2035
8	North Bay, Timmins, Driftwood	1162	2052
9	Sudbury, Matheson, Driftwood	1199	2055
10	Sudbury, Timmins, Smooth Rock Falls	1268	2074
11	North Bay, Driftwood, Cochrane	1231	2085
12	North Bay, Timmins, Smooth Rock Falls	1162	2104
13	Barrie, North Bay Matheson	981	2107
13	Sudbury, Matheson, Smooth Rock Falls	1199	2107
14	North Bay, Timmins, Cochrane	1224	2114
15	Sudbury, North Bay, Driftwood	1200	2128
16	Sudbury, Kirkland Lake, Driftwood	1208	2136
17	North Bay, Cochrane, Smooth Rock Falls	1231	2137
18	North Bay, Timmins, Matheson	1376	2146

Considering the rankings in Table 21, triples containing North Bay and Matheson figure strongly in these rankings. Recalling that the pair North Bay, Matheson ranked highest for both the 2-center and the modified 2-center problems, such a result in the triples only serves to strengthen the conclusion that Matheson is a viable candidate for a depot, whether it be replacing Sudbury or as an added third depot to the existing situation. In fact, in considering both the median and modified median rankings for the 3-median scenario, the triple Barrie, North Bay, Matheson has the 5$^{\text{th}}$ lowest median and the 13$^{\text{th}}$ lowest modified median making this triple the best overall choice in the three median scenario.

10 Developing a New Schedule

To demonstrate the value of the findings of this paper, we will now explore the notion that moving one of the current depots to another location as dictated by this research will improve the overall efficiency of the busing operations conducted by Ontario Northland. After finding that the pair North Bay, Matheson is not only optimal for the 2-center and 2-median considerations but also figures often in the top results for the 3-center and 3-median scenarios, we will explore the benefits of moving the depot that is currently located in Sudbury to Matheson. To do this, we developed a schedule with the depots located at North Bay and Matheson which we compared to the current schedule to demonstrate that it is at least as efficient, if not more so, than the current.

10.1 Scheduling Considerations

It is important to note before further discussion regarding the proposed schedule that there are multiple factors influencing scheduling - passenger density, driver's working requirements, shipping considerations etc; however, our example is derived based only on the geometric considerations (the 2-center and the 2-median) discussed in the previous sections.

Unfortunately, as mentioned already in the paper, ONTC is not authorized to run buses from North Bay to Sudbury because this route belongs to another bus company. In light of this restriction, it becomes necessary to run a bus that is not based out of North Bay or Matheson in order to connect Sudbury to Toronto. The alternative to this is to run buses in a circuit from North Bay to Toronto and then up to Sudbury but then ONTC would have to pay their drivers for them to make the 2hr trip home from Sudbury to North Bay after their route ended. ONTC would essentially be running empty buses along the North Bay, Sudbury route which clearly is not very desirable. Therefore, we will run buses from Toronto to Sudbury and back in the proposed schedule and then we will explore the possibilities for the circuit routes discussed above.

10.2 The Proposed Schedule

The first important aspect of the new schedule is to note that it has as many or more runs daily from each city to each other city as the current schedule with one exception: Cochrane to Driftwood in the current schedule runs 3 times daily while in the proposed schedule it runs twice.(This is not a major exception and could likely be remedied if ONTC were to insist on the necessity of 3 runs per day). Thus, the service offered by this schedule is very closely equivalent to that offered by the current schedule.

ONTC requires a minimum of 30min between connections [1]. A careful examination of the proposed schedule will reveal that many of the connections are optimized; in other words, with the proposed schedule passengers rarely are required to wait more than 30 to 45mins to connect to the next bus on their route. Also, our proposed schedule accommodates well the requirements that drivers have 14hr workdays with no more than 13hrs driving [1]. For example, the proposed schedule with depots at North Bay and Matheson is configured so that bus drivers can easily drive to and from their destinations with little wait time in between routes; in other words, the driver of the 701S route from North Bay to Toronto has a 15min break in Gravenhurst on his/her way South to Toronto. Once that driver arrives in Toronto, he/she has a 30min break before heading back to North Bay driving the 701N route and again having a 15min break in Gravenhurst. In this way, the driver receives their total required break time of 1hr and works about a 12hr day. Similarly, the drivers who travel the 702S, 703S and 704S routes return after a short break on the 702N, 703N and 704N routes respectively. All the routes, with the exception of 501N/S and 502N/S, are set up in this way - the driver leaves the depot on a specifically numbered route in one direction and returns to the depot driving the route of the same number in the opposite direction.

The shuttle service is also well configured in the proposed schedule to allow passengers to travel easily from Matheson to either Cochrane, Driftwood or Timmins after arriving in Matheson from another city such as Timmins, Kirkland Lake or North Bay. For example, a passenger travelling from Driftwood to North Bay with the current schedule would leave Driftwood at 07:25 and arrive in North Bay at 15:00 - a total travel time of 7hrs 35 mins. With the proposed schedule a passenger would simply take the SR1c at 08:35 to arrive in Matheson at 10:15 and then at 10:50 he/she would take the 501S to arrive in North Bay at 14:55 with a total travel time of 6hrs 20mins; thus, the passenger would save over an hour of travel time with the proposed schedule due to the connectedness of its shuttle service to the rest of the services offered by ONTC.

Table 22: North Bay $\Rightarrow$ Toronto

Trip-Run	701S	702S	703S	704S
North Bay	13:00	15:30	00:30	06:00
Barrie	17:05	19:35	04:35	10:05
Toronto	18:35	21:05	06:05	11:35

Table 23: Toronto $\Rightarrow$ North Bay

Trip-Run	701N	702N	703N	704N
Toronto	19:05	23:30	06:35	12:10
Barrie	20:35	01:00	08:05	13:40
North Bay	00:40	05:05	12:10	17:45

Table 24: North Bay $\Rightarrow$ Matheson

Trip-Run	501N	502N
North Bay	12:50	18:15
Kirkland Lake	16:30	21:55
Matheson	17:35	23:00

Table 25: Matheson $\Rightarrow$ North Bay

Trip-Run	501S	502S
Matheson	10:50	20:25
Kirkland Lake	11:15	20:50
North Bay	14:55	01:30

Table 26: Toronto ⇒ Sudbury

Trip-Run	101N	102N	103N
Toronto	08:00	14:30	18:00
Barrie	09:30	16:00	19:30
Sudbury	13:45	20:15	23:45

Table 27: Sudbury ⇒ Toronto

Trip-Run	101S	102S	103S
Sudbury	14:30	08:00	19:00
Barrie	18:45	12:15	23:15
Toronto	20:15	13:45	00:45

Table 28: Matheson ⇒ Sudbury

Trip-Run	801S
Matheson	08:45
Timmins	09:40
Sudbury	13:55

Table 29: Sudbury ⇒ Matheson

Trip-Run	801N
Sudbury	14:20
Timmins	18:35
Matheson	20:25

Table 30: Matheson ⇒ Hearst

Trip-Run	901N
Matheson	18:05
Timmins	19:15
Driftwood	20:10
Smooth Rock Falls	20:30
Hearst	22:45

Table 31: Hearst ⇒ Matheson

Trip-Run	901S
Hearst	05:40
Smooth Rock Falls	07:55
Driftwood	08:15
Timmins	09:10
Matheson	10:20

Table 32: Shuttle Clockwise: Matheson ⇒ Timmins ⇒ Driftwood ⇒ Cochrane ⇒ Matheson

Trip-Run	SR1c	SR2c
Matheson	06:30	17:50
Timmins	07:40	19:00
Driftwood	08:35	19:55
Cochrane	09:00	20:20
Matheson	10:15	21:35

Table 33: Shuttle Counterclockwise: Matheson ⇒ Cochrane ⇒ Driftwood ⇒ Timmins ⇒ Matheson

Trip-Run	SR1cc	SR2cc
Matheson	11:00	18:00
Cochrane	12:15	19:15
Driftwood	13:40	19:40
Timmins	14:35	20:35
Matheson	15:45	21:45

10.3 Comparing Travel Time and Routes Between the Current Schedule and the Proposed Schedule

The proposed schedule with depots at North Bay and Matheson as opposed to North Bay and Sudbury shortens some of the overall travel time between certain cities while rarely lengthening the travel time between others. For example, the overall travel time from Toronto to Hearst through North Bay with the current schedule is 16hrs 45mins. With the new schedule a passenger could leave Toronto on route 703N at 06:35 and have a 40min transfer in North Bay for route 501N to Matheson. Then, from Matheson after a 30min transfer the passenger would take route 901N and arrive in Hearst at 22:45; thus, the total travel time with the new schedule from Toronto to Hearst via North Bay is 16hrs 10mins - a 35 minute improvement over the current situation.

Figure 2: Proposed Schedule Displayed on Map (Some departure times not shown)

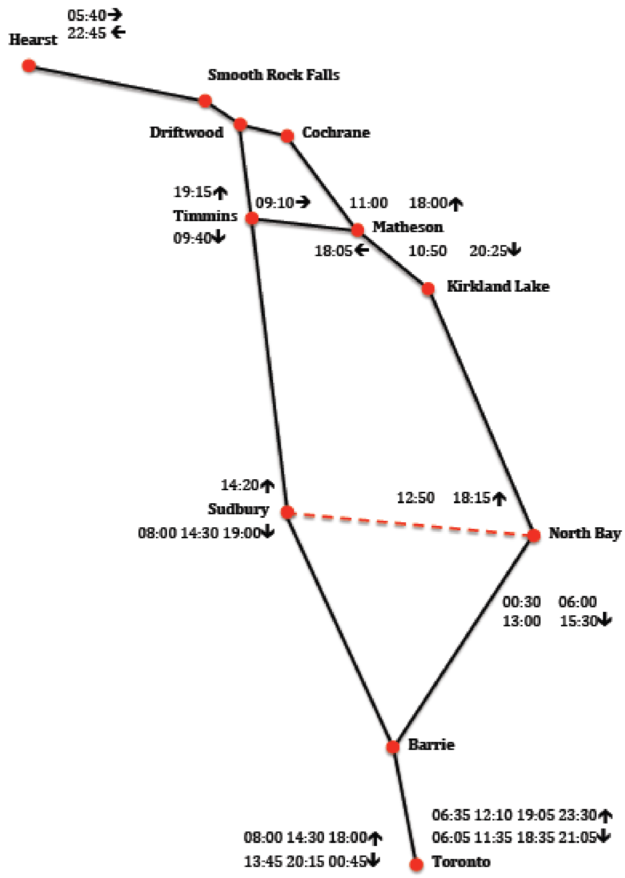
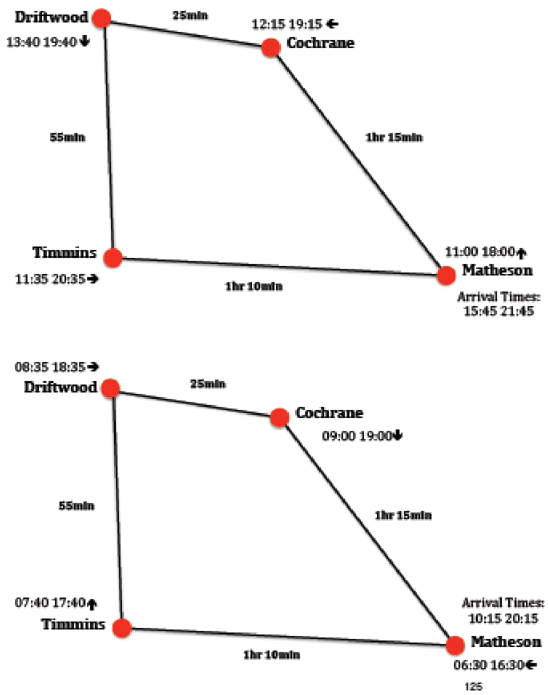


Figure 3: Proposed Shuttle Schedule Displayed on Map



Not only is the total travel time between these two extreme points of the ONTC map significantly reduced, the current schedule has an arrival time of 02:00 in Hearst while the proposed schedule allows the passenger to make the trip in one day and to arrive at a slightly more reasonable time (both for the convenience of the passenger and the bus driver).

On the return trip from Hearst to Toronto through North Bay, the proposed schedule would also shorten the travel time. The passenger would board the bus at 05:40 in Hearst on route 901S, travel straight to Matheson (no transfer in Timmins as there was on the trip North), have a half hour transfer in Matheson and then take the 501S to North Bay. Then they would have a 35min transfer in North Bay to arrive in Toronto on the 702S route at 21:05. That is a total travel time with the new schedule of 15hrs 25mins. Travelling in the current system, a passenger would have to leave Hearst at 05:00, have a 25min transfer in Timmins, then an hour transfer in North Bay with a total travel time of 16hrs 45mins. Thus, the current schedule results in an overall time savings for a passenger travelling Hearst to Toronto through North Bay of 1hr 20mins. Clearly, this is a significant improvement over the current situation.

Similarly, travelling from Toronto to Hearst through Sudbury with the proposed schedule takes 14hrs 45mins - 15 minutes less time than with the current schedule (Note: a passenger travelling from Toronto to Hearst would typically choose to take this route through Sudbury as opposed to the route that travels through North Bay for the simple reason that the travel time is much shorter). A passenger travelling from Toronto to Hearst in the proposed system with depots at North Bay and Matheson would begin their trip at 08:00 in Toronto on the 101N bus. The passenger would then have a 45min transfer in Sudbury and a 40 min transfer in Timmins to arrive in Hearst at 22:45. With the current schedule that same passenger would have to leave Toronto at 11:00, have a 45min transfer in Sudbury, a 45min transfer in Timmins and arrive in Hearst at 02:00. This current route has a total travel time of 15hrs - 15min longer than the route provided by the new schedule with depots at North Bay and Matheson.

Returning to Toronto from Hearst through Sudbury, the passenger in the proposed system would leave Hearst at 05:40 on 901S, have a half an hour transfer in Timmins, catch the 801S route as it travelled through Timmins to Sudbury. From Sudbury, the passenger would have a 35min wait for the 101S and would arrive in Toronto at 20:15 - a total travel time of 14hrs 35mins. With the depots at North Bay and Sudbury and the current schedule, this passenger leaves Hearst at 05:00, has over a 3 hour transfer in Timmins, and then a 35min transfer in Sudbury to arrive in Toronto at 22:15 resulting in a total travel time of 17hrs 15mins; therefore, with the proposed schedule a passenger could make the trip from Hearst to Toronto through Sudbury in 2hrs 40mins less time than with the current schedule - a very significant reduction in overall travel time. The benefits of the knowledge gained from considering the geometric layout of the system map (even with no computation capabilities) is undeniable.

As was previously mentioned, the overall travel time between Toronto and Hearst (the two extreme points of the service map) is reduced with the proposed schedule. In addition to this route, there are many other routes whose overall travel time is lessened with the proposed schedule. For example, travelling from Toronto to Matheson with the current schedule takes a total of 11hrs 45mins while with the proposed schedule it takes only 11hrs. A passenger travelling from Toronto to Matheson currently would have to wait 1hr 25mins for a transfer in North Bay. With the proposed schedule, this wait time is only 40mins. Considering that ONTC requires the wait time between transfers to be at least 30mins to try compensate for unknown factors involved in driving such as construction, accidents, weather conditions etc. and to allow drivers to have the required amount of time for their breaks, a 40min transfer is very reasonable.

Displayed in Table 34 is a summary of 12 routes and their overall travel times with the current schedule and then with the proposed schedule. The time savings with the proposed schedule are anywhere from 5mins (in the case of Sudbury to Driftwood) to almost 3hrs (the run from Hearst to Toronto via Sudbury). The only run displayed in Table 34 for which it takes more time to complete with the proposed schedule is North Bay to Driftwood. Taking into account the fact that Matheson is a depot in the proposed schedule and thus must have at least a half hour transfer that is not present in the current schedule, an extra 10mins in overall travel time is both a negligible amount of time and a small price to pay for the other immense time gains that result from the proposed schedule.

Table 34: Proposed Schedule Travel Time Benefits

Route	Current Time	Time with Proposed Schedule	Time Savings
Toronto - Matheson	11:45	11:00	00:45
Matheson - Toronto	11:45	10:15	01:30
Toronto - Timmins	12:55	10:35	02:20
Timmins - Toronto	10:45	10:35	00:10
Toronto - Hearst (via North Bay)	16:45	16:10	00:35
Hearst - Toronto (via North Bay)	16:45	15:25	01:20
Toronto - Hearst (via Sudbury)	14:45	14:30	00:15
Hearst - Toronto (via Sudbury)	17:15	14:35	02:40
North Bay - Driftwood	07:10	07:20	-00:10
Driftwood - North Bay	07:35	06:20	01:15
Sudbury - Driftwood	05:55	05:50	00:05
Driftwood - Sudbury	08:30	07:40	00:50

10.4 Other Considerations/Options: Passenger Density and Circular Routes

Clearly, the benefits of studying the geometric layout of the ONTC bus network are many - shorter overall travel times for passengers, more convenient and shorter transfers, better connected routes, more efficient use of driver time paid etc. The absence of the option to run buses from North Bay to Sudbury necessitated a practical adjustment to the theoretical geometric findings; in other words, because we cannot run buses along this route, we did not develop a purely 2-center model in light of the fact that we had to run buses out of Sudbury (the Sudbury to Toronto routes) as well as out of our two depots, North Bay and Matheson. Of course, we have the option to allow drivers based in North Bay to commute to and from Sudbury in order to run these routes but this course of action would probably necessitate the compensation of drivers for over 4hrs/shift of time spent driving empty buses. Also, these extra 4hrs driving per shift would put drivers over the maximum allowed driving time of 13hrs/day; thus, this is not a viable or cost-effective option.

Nevertheless, we considered the possibility of running buses in a loop from North Bay to Toronto to Sudbury and then back to North Bay and of course in the other direction: North Bay to Sudbury to Toronto and back to North Bay. In this way, drivers could be based in North Bay and although we would still be running empty buses, the loop would mean that we would run the buses between North Bay and Sudbury only once per driver shift (as opposed to the option of running these buses from North Bay to Sudbury, then down to Toronto, back to Sudbury and then back to North Bay, essentially travelling between North Bay and Sudbury twice in one shift). This modification would indeed allow our model to become a purely 2-center model with buses running only out of the depots at North Bay and Matheson.

Keeping the other routes of the schedule the same, we modified the times of the routes between North Bay and Toronto and between Sudbury and Toronto to create the aforementioned loops. In the current schedule, as in the proposed schedule, there are four daily runs from North Bay to Toronto and back as well as three daily runs from Sudbury to Toronto and back; therefore, because there is one more route required daily from North Bay to Toronto and back, we will need three loops and one route which simply runs North Bay to Toronto and then returns to North Bay. Table 35 and Table 36 display a rough schedule for these circular routes. The routes in Tabe 35 have the driver begin their shift in North Bay, travel to Toronto where they would have at least a half hour break, then on to Sudbury and then travel home to North Bay with the empty bus (this arrival time home to North Bay is not displayed because it would not be on the passenger schedule as passengers would not be allowed to ride the bus from Sudbury to North Bay). Similarly, in Table 36, the driver would drive an empty bus from North Bay to Sudbury at the start of their shift, then head South with passengers to Toronto and arrive home with passengers on board in North Bay to end their shift. Table 37 simply displays the one route, as mentioned above, that would run from North Bay to Toronto and back to North Bay without going to Sudbury.

Although the times in these schedules may not seem as well spread out or convenient as those in the proposed schedule with no circular runs, this schedule is set up so that the connections with other cities from the original proposed schedule are preserved. In other words, passengers travelling in the schedule with circular routes would not find it much more difficult or inconvenient to travel beyond Sudbury and North Bay or to return to Toronto from locations further North than Sudbury and North Bay.

Table 35: Clockwise Circular Routes: North Bay ⇒ Toronto ⇒ Sudbury ⇒ North Bay

Trip-Run	101c	102c	103c
North Bay	13:00	02:00	18:10
Barrie	17:05	06:05	22:15
Toronto	18:35 - 19:10	07:35 - 08:00	23:45 - 00:30
Barrie	20:40	09:30	02:00
Sudbury	00:55	13:45	06:15

Table 36: Counterclockwise Circular Routes: North Bay ⇒ Sudbury ⇒ Toronto ⇒ North Bay

Trip-Run	101cc	102cc	103cc
Sudbury	14:30	12:00	09:00
Barrie	18:45	16:15	13:15
Toronto	20:15 - 20:45	17:45 - 18:30	13:45 - 14:30
Barrie	22:15	20:00	16:00
North Bay	02:20	00:05	20:05

Table 37: Route for Circular Option: North Bay ⇒ Toronto ⇒ North Bay

Trip-Run	701
North Bay	00:30

Barrie	04:35
Toronto	06:05 - 06:35
Barrie	08:05
North Bay	12:10

Interestingly, the limited passenger count data provided by ONTC from January to June 2011 suggests that only two circular runs are necessary with another two runs that simply run from North Bay to Toronto and back without going to Sudbury. A cursory glance at the data immediately suggests that the ratio of passengers travelling between Sudbury and Toronto to those travelling between North Bay and Toronto is 1500:4000 or approximately 3:8; however, ONTC currently runs three daily routes between Sudbury and Toronto and four daily routes between North Bay and Toronto, suggesting a passenger ratio of 3:4 - much higher than the actual figure. Such a significant observation made from a very small sample of data and without any computation at all clearly suggests that the passenger density data needs to be extensively studied separately to improve the efficiency of the ONTC bussing operation; indeed, this large discrepancy from what the data clearly suggests and what is actually being done demonstrates the desperate need for more in depth study of the passenger data.

11 Discussion about the Limitations of this Study and Future Work

The paper presents a case study, aimed at suggesting better and more efficient ways to operate transit services in Northern Ontario by the crown corporation which currently offers (exclusively!) these services - Ontario Northland. It is clearly said that this case study focuses on only one aspect of the operations, namely the location(s) of the depot(s). Further, it only considers three possible scenarios - one, two or three bus depots. We are aware that the problem has more dimensions, and that normally a multi-objective optimization would be required. It is clearly stated in the paper that this is just one step in approaching the problem systematically and developing a software tool that will be able to address all the aspects of the problem - customer density and waiting times, drivers scheduling, bus scheduling (i.e. which physical vehicle services which route on which day and time), scheduling of parcels, etc. The findings in the paper clearly demonstrate that just studying this problem from one (may it be narrow) point of view - the geometry of the network - leads to the possibility of a better operation through relocation of depots, and that significantly better schedule is possible in terms of waiting times, transfer times and total time for a trip between two locations. These findings are part of ongoing research. They are also part of ongoing funding applications to obtain financing which will allow hiring appropriate personnel to continue the project. The k-median/k-center models were chosen, as stated in the paper because these are the most natural models when it comes to distances only, or when everything is directly proportional to the distances in the network. We are mindful of the fact, and it is clearly mentioned in the conclusion, that weighted models/approaches may be employed later to make the model more precise.

We are not aware of other sources that use the same approach for bus scheduling. Again, as mentioned above, usually it is a multi-faceted problem, so it cannot be completely addressed through the geometry of the road network only. However, given the data we received from Ontario Northland, it was glaring that immediate improvements can be achieved if the depots are located appropriately, i.e. where they should be if economic/business reasons are used as opposed to historic/political reasons. We do not expect that the approach is novel, quite the opposite - this is a basic technique in facility location. What is the most original aspect of it? Probably the fact that in more than 50 years nobody took a look at this from such a simple point of view, and nobody was willing to even consider it. Although we are not at all concerned in applying the same method or argument to larger scale problem, the answer is that it is applicable in the ways and in combination with other factors, as already discussed. We study a particular operation, of particular size, at specific geographic location and in the current economic and political environment in Ontario.

The sites in the network are not weighted or equally weighted. The passenger demand is not included in the model. It would be included in a more complex model. However, since this is a government-run operation, the primary concern is coverage, i.e. the routes will be run even without passengers (moreover, we were told by Ontario Northland that it really is the case on the Northernmost routes very frequently). We clearly discuss in the paper the disproportionate number of trips between Sudbury and Toronto and between North Bay and Toronto, which does not follow the passengers' numbers. The waiting times for the passengers are not included in the model. However, the alternative schedule we provide improves on the transfer times (i.e. wait between consecutive rides) and the overall time to the destination.

12 Conclusion

The benefits of studying the geometric layout of the ONTC bussing network are undeniably evident. Not only were we able to determine a much more efficient set up for the depots utilizing the 2-center and 2-median models, we were able to determine the optimal depot locations for a system with 3 depots as well and we deduced all of this valuable information without any computational aid; in fact, we developed a schedule by hand based on the findings of these geometric models that was clearly shown to be more efficient, cost-effective and convenient for both passengers and ONTC itself.

The next logical step in this process would be to begin developing a software program based on the k-center and k-median models that could not only determine optimal positions for depots given a map of the bussing network, but that could also generate a new schedule utilizing these depot locations and be able to compare such a schedule to the current or another suggested schedule. Also, as was mentioned briefly above, the passenger density should be extensively studied and it will yield a different (weighted) model which should then be incorporated into the customized software tool along with the standard (linear algebra/optimization) models/approaches that will reflect inventory constraints (i.e.. buses, drivers, shifts etc). The software would be capable of designing a schedule as well as evaluating a schedule on a number of criteria (with improvement suggestions) and then comparing two or more schedules to one another; again, the benefits and uses of such a software tool for both ONTC and many other transportation networks are great in both number and scope.

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Competing Interests

The authors declare that no competing interests exist.

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