Residual Control Rights, Transferable Returns, and Their Implications for Ownership Structure

Meri Melkonyan
Prince George's Community College, Largo, USA
Email: melkonmx@pgcc.edu

ABSTRACT

The paper examines the consolidation problem in a model characterized by non-contractible, relationship specific investments, transferable and non-transferable payoffs, and ex post actions that are chosen after the uncertainty in the model is realized. We determine the relationship between the optimal ownership structure and the nature of the relationship specific investments and ex post actions and the degree to which payoffs are transferable with ownership.

1. Introduction

Consolidation is a growing trend in both established and relatively new industries, e.g. automobiles, electronics, biotechnology, retailing, telecommunications, and transportation. Explanations for the trends in consolidation include economies of scale and scope, exercise of monopoly power, informational asymmetries, and contractual incompleteness. Our analysis of consolidation builds on the existing incomplete contracts and residual property rights results. Specifically, the model structure is motivated by the results of [1,2] on the distribution of residual property rights. We focus on the non-contractibility of "managerial services" and the relationship specificity of investments. Managers make relationship specific investment decisions and agreements to exchange ownership. The incentives for exchange of ownership are provided by the payoffs associated with the relationship specificity of the investments. Investment levels are influenced by the contractual incompleteness.

Our model is further distinguished by the structure of the payoffs. Managers may have incentives to invest that relate to their ownership of the shares in the two firms, if there is a consolidation. Managers also have incentives that relate to payoffs that only they can appropriate. Examples of the latter include credibility of the manager, special creative and other abilities of the managers, the disutility of taking certain actions, job satisfaction, etc. Our results provide an opportunity for understanding interactions between these private payoffs and those associated with ownership of shares or the "transferable payoff" from the consolidated enterprise. Thus, the ownership structure influences both the transferable payoff and the private incentives of the managers.

We distinguish between two types of "cooperative" relationship specific investments [3]. In one case, the investments are complementary. That is, the marginal value of one manager's investment increases with the level of the other manager's investment. In the other case, the investments are substitutes. That is, the marginal value of one manager's investment decreases with the level of the other manager's investment. Incentives for consolidation depend on whether investments are strategic complements or strategic substitutes. The degree of complementarity or substitutability of the actions that are taken after investments are made also affects incentives of the managers to invest and the optimal ownership structure.

A unique aspect of our results is the capacity to identify optimal ownership shares in the case of consolidation. Managers may have incentives to consolidate as a way of mitigating the investment inefficiencies of incomplete contracts. First, we identify the first- and second-best solutions to the related game. The second-best solutions can be intuitively characterized using standard economic tools. Second, we generalize these results on consolidation by introducing control rights which affect the optimal ownership structure and associated investments and actions of the managers.

2. The Basic Model and the First-Best Solution

We consider a four stage game between two risk-neutral players. For the ease of presentation we assume that before the game is played the two players own and manage two separate firms, 1 and 2[1]. Henceforth, the two players are called managers M_1 and M_2. Ownership of a firm gives the managers claim to the assets of the firm and associated returns (also called transferable returns) and decision rights over the choice of "some" of the actions that are made by the firm. As will be clear later, the manager who is the majority owner has control over the use of the assets of the firm but not necessarily the control over the manager who owns the minority of the shares.

The assumed sequence of actions of the managers is shown in **Figure 1**. Period t_0 is the contracting date. This may include negotiations between the managers that result in a different ownership structure of the two firms. We allow for the possibility that the firms can be jointly owned. In most of the following analysis we concentrate on the exchanges of voting equity between the two managers. Thus, if we let s_i denote the share of firm i's voting equity (henceforth simply called shares) owned by manager M_1, she/he receives portion s_i of transferable returns of firm i and, in the case when $s_i > 1/2$, has full control over the future actions using the assets of firm i. We consider three qualitatively different possibilities for the ownership structure; M_1 has a majority ownership of firm 1 and M_2 has a majority ownership of firm 2, M_1 has a majority ownership of both firms, and M_2 has a majority ownership of both firms. We do not consider the case in which M_1 has a majority ownership of firm 2 and M_2 has a majority ownership of firm 1. We assume that this ownership structure is less efficient than the one in which M_1 has complete ownership of firm 1 and M_2 has complete ownership of firm 2.

In period t_1, given the ownership structure chosen in period t_0 and possibly a more complicated contract governing the future relationship between the two parties, the two managers choose investments that affect the future potential gains from the relationship between the two firms. We let e_j denote the level and the cost of investment by manager M_j ($j = 1, 2$). We model this by assuming that these investments (e_1, e_2) affect the probability of the state of nature $\theta \in \Theta$, where Θ is the set of possible states of nature. The probability of the state of nature $\theta \in \Theta$ given investments (e_1, e_2) is denoted by $p_\theta(e_1, e_2)$. For a research and development project, a state of nature may be characterized by the occurrence of a scientific discovery ([4]). More generally, a state of nature may reflect the future profitability of a firm ([5]).

Both managers learn the realization of the state of nature in the beginning of period t_2. In period t_3 actions a_1 and a_2 are chosen. For example, in a research and development context, a_1 and a_2 could reflect a decision on the development effort. Given the realization of the state of nature θ and the choice of actions (a_1, a_2) in period t_3, the ex post payoff function (after the investments are made and the state of nature is realized) of manager M_1 is given by

$$U_1\left(a_1, a_2, T, \theta \mid s_1, s_2\right) = s_1 u_1^T\left(a_1, a_2, \theta\right) + s_2 u_2^T\left(a_1, a_2, \theta\right) + u_1^P\left(a_1, a_2, \theta\right) + T, \tag{1}$$

where $u_i^T\left(a_1, a_2, \theta\right)$ is the transferable return to the owner of firm $i = 1, 2$, $u_1^P\left(a_1, a_2, \theta\right)$ is the private or non-transferable benefit to the manager of firm 1, and T is the monetary transfer from manager M_2 to manager M_1. Note from the structure of the model that manager M_i ($i = 1, 2$) receives private benefit that is influenced by both investments and actions of the two managers. Investments e_1 and e_2 affect private benefits indirectly through their effect on the realization of the state of nature θ, while actions a_1 and a_2 affect them directly.
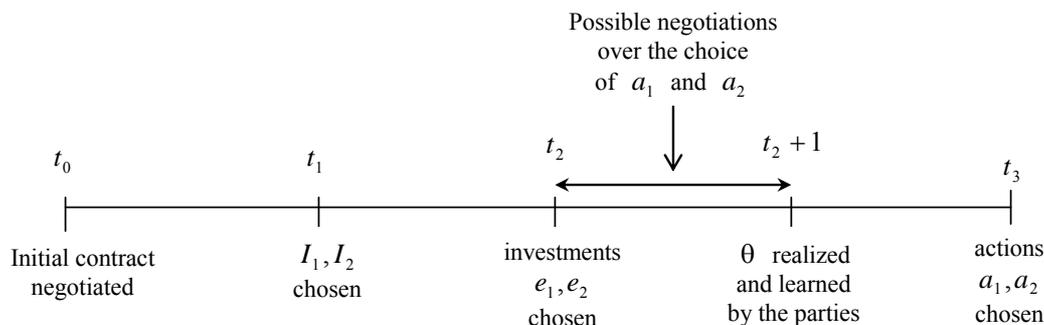

Figure 1. Timing of events.

Similarly manager M_2's ex post payoff can be written as

$$U_2\left(a_1, a_2, T, \theta \mid s_1, s_2\right)$$
$$= (1 - s_1) u_1^T (a_1, a_2, \theta) + (1 - s_2) u_2^T (a_1, a_2, \theta) \quad (2)$$
$$+ u_2^P (a_1, a_2, \theta) - T$$

where $u_2^P (a_1, a_2, \theta)$ is the private or non-transferable benefit to the manager of firm 2.

The total ex post surplus is given by

$$S(a_1, a_2, \theta) = u_1^T (a_1, a_2, \theta) + u_2^T (a_1, a_2, \theta)$$
$$+ u_1^P (a_1, a_2, \theta) + u_2^P (a_1, a_2, \theta) \quad (3)$$

The first-best action choices[2] $\left(a_1^*(\theta), a_2^*(\theta)\right)$ for $\theta \in \Theta$ maximize the total ex post surplus and are given by the following system of equations

$$\frac{\partial u_1^T(a_1^*(\theta), a_2^*(\theta), \theta)}{\partial a_i} + \frac{\partial u_2^T(a_1^*(\theta), a_2^*(\theta), \theta)}{\partial a_i}$$
$$+ \frac{\partial u_1^P(a_1^*(\theta), a_2^*(\theta), \theta)}{\partial a_i} + \frac{\partial u_2^P(a_1^*(\theta), a_2^*(\theta), \theta)}{\partial a_i} = 0 \quad (4)$$

for $i = 1, 2$.

The first-best investments (e_1^*, e_2^*) maximize the ex ante total surplus, given the first-best action levels for each realization of the state of nature

$$(e_1^*, e_2^*)$$
$$= \arg\max_{e_1, e_2} \sum_{\theta \in \Theta} p_\theta(e_1, e_2) S(a_1^*(\theta), a_2^*(\theta), \theta) - e_1 - e_2 \quad (5)$$

These first-best investment levels satisfy the following system of equations

$$\sum_{\theta \in \Theta} \frac{\partial p_\theta(e_1^*, e_2^*)}{\partial e_i} S(a_1^*(\theta), a_2^*(\theta), \theta) = 1 \text{ for } i = 1, 2.$$
$$(6)$$

Since the information at the beginning of stage t_2 is complete, the ex post negotiation between t_2 and $t_2 + 1$ will always lead to the first-best actions $\left(a_1^*(\theta), a_2^*(\theta)\right)$ being chosen in stage t_3 for each possible state of nature θ. Thus, if the two parties could write a contract that would specify the investment levels (e_1^*, e_2^*), the first-best would be implemented.

3. Contracting

If the two managers could contract on the investments e_1 and e_2 they would choose them to maximize the total ex ante surplus and would distribute this surplus via ex ante lump sum transfers.[3] In this kind of environment the

allocation of ownership (who owns assets of firms 1 and 2) is irrelevant. This observation is consistent with arguments by [1,6], among others, who observe that when it is costless to include all relevant contingencies in a contract, and these contingencies can be foreseen, the allocation of property rights is indeterminate. Thus, to generate non-trivial predictions about optimal ownership structure one must consider scenarios where some aspects of the environment can not be specified in a contract. This is the route that we are taking in our paper.

In what follows we assume that investments e_1 and e_2 are not verifiable to third parties and, hence, can not be included in a contract. In the example of research and development projects, these investments may stand for the time and effort a manager spends working on a project, his/her creativity. It is very hard to find an objective measure of this type of investment. We assume that the ex post actions, a_1 and a_2, are ex post verifiable. Thus, given investment levels e_1 and e_2, the two managers will choose a_1 and a_2 to maximize the total ex post surplus for all states of nature $\theta \in \Theta$.

The two parties may (and will) benefit (relative to having no contract at an ex ante stage) if they could devise a game form (also referred to as a mechanism and a message game) such that the ex post actions and monetary transfers were functions of the two players' strategies. This would allow for the modification of the division of the ex post surplus in each state of nature and hence would alter the investment incentives of the two managers. Suppose that the two parties want to achieve a particular division of the ex post surplus for each state of nature. The question is whether there is a message game that can indirectly implement a particular choice rule, in our case the assignment of actions and transfer payments for each state of nature.

This question was first addressed by [7] who identified the set of social choice rules that are implementable when the contracting parties have symmetric information but when this information is not verifiable to third parties[4,5]. His ingenious mechanism utilizes nuisance strategies to get rid of unwanted equilibria. However, Maskin's mechanism is not renegotiation proof in a sense that it may result in non-optimal equilibria off the equilibrium path. The ability of the contracting parties to renegotiate the outcome of the message game constrains the set of choice rules that can be implemented. [10] characterizes implementable social choice rules for which the parties can not commit not to renegotiate the outcome of the

---

[2] In what follows we assume unique and interior solutions for all optimization problems.

[3] The magnitude of these transfer payments would be determined by the relative bargaining powers of the two managers.

[4] [7] shows that social choice rules satisfying monotonicity and no veto power conditions are implementable in Nash equilibrium provided there are at least three players. [8] considers implementation in subgame perfect equilibrium and finds that a larger set of social choice rules can be implemented.

[5] For an excellent source on complete information implementation, see [9].

message game.

On the other hand, if the contracting environment is sufficiently complex in the sense that there are many possible contingencies of θ to consider and/or there is an exogenous cost of including additional contingencies in a contract, then the value of having a message game over no-contract is almost nil ([11]). Also, when the ex ante investments are cooperative in nature (that is, they affect both own payoff and payoff of the other party), the ability of writing an ex ante message game may not improve upon no contract under a wide range of scenarios.

In what follows we consider alternative contracting environments. In all cases we assume that the contracting parties are unable to commit not to renegotiate the inefficient outcomes of the message game. Also, recall that we have assumed that the ex post actions are ex post contractible. First, we consider a situation where the states of nature are describable ex ante, and then derive the optimal contract. We also consider the implications of indescribability of the states of nature. Second, we allow for the message game but impose a certain structure on it. In particular, we consider only message games where ownership shares and transfer payments are contingent on the players' strategies. Third, we employ the assumption that the message game cannot be used in stage t_2. We find the optimal ownership allocation for this case and the resulting investment levels.

4. Complete Contract When Future Physical Contingencies Are Describable[6]

As a benchmark case we consider a situation where the state of the world is verifiable ex post (after investments e_1 and e_2 are made) and is describable ex ante[7]. Under these assumptions rational players can write a complete contract that specifies actions to be chosen in each state of nature. That is, a complete contract is a function $f: \Theta \to A_1 \times A_2 \times Y$. Since lump sum transfers are allowed at both ex ante and ex post stages, the contracting parties will choose a complete contract that is constrained Pareto optimal. That is, the actions specified by f are $\left(a_1^*(\theta), a_2^*(\theta) \right)$ for all $\theta \in \Theta$. Given contract f player i's payoff is

$$\sum_{\theta \in \Theta} p_\theta(e_1, e_2) U_i(f(\theta), \theta) - e_i \qquad (7)$$

Thus, the contract f induces a game in which the players simultaneously and independently choose investments e_1 and e_2 and players i's payoff is given by (7).

The pair $\left(e_i^c, f \right)$ is feasible if, given f, the unique equilibrium of the investment game consists of each

agent i choosing $e_i = e_i^c$:

$$e_i^c = \arg\max_{e_i} \left(\sum_{\theta \in \Theta} p_\theta(e_1, e_2) U_i(f(\theta), \theta) - e_i \right) \quad (8)$$

Thus, to find the second-best investment levels one has to solve a moral hazard problem with two agents[8]. This multi-person moral hazard problem can be written as

$$\max_{\lambda(\bullet), e_1, e_2} \sum_{\theta \in \Theta} p_\theta(e_1, e_2) S\left(a_1^*(\theta), a_2^*(\theta), \theta \right) - e_1 - e_2 \quad (9)$$

s.t. $e_i = \arg\max_{e_i} \left(\sum_{\theta \in \Theta} p_\theta(e_1, e_2) \left(\lambda(\theta) S(\theta) \right) - e_i \right)$.

One can easily see from this optimization problem that the solution to this moral hazard problem can be realized as a state-contingent exchange of ownership. One can interpret this result as a complete contract foundation for optimal ownership. [12,13] make a similar type of argument.

Now suppose that the state of nature is not describable ex ante. [12] shows that indescribability does not interfere with optimal contracting as long as the parties can commit not to renegotiate the message game. They also show that even if renegotiation is allowed the above result holds as long as the parties are risk averse. Note that we have assumed that the parties are risk neutral and, hence, indescribability will constrain the set of social choice rules that are implementable.

5. Simple Contract

In this section we consider a situation where at the contracting date t_0 the two managers M_1 and M_2 can only write a contract that specifies an exchange of ownership and possible lump-sum transfer payments. That is, we rule out the possibility of the contracting parties devising a message game that will be played at a later date t_2. Consideration of this environment can be justified by assuming that even if the contracting parties can devise a mechanism that would specify outcomes (in our case, actions a_1 and a_2 and lump-sum transfer payments between the two parties) as functions of the two parties' strategies in the message game, they cannot prevent renegotiation of inefficient outcomes off the equilibrium path of the mechanism.[9] If the future contingencies are indescribable (that is, the contracting parties can not describe the possible states of nature in advance), then these two assumptions (coupled with the risk neutrality assumed in our model) constrain the set of payoffs that can be reached under a complete contract ([12]).

Thus, the simple contract is a triple (s_1, s_2, t) where t is a transfer payment of manager M_1 to manager M_2. In these circumstances the ownership will affect the in-

---

[6]Our discussion and definitions in this section closely follow that of [12].
[7]We assume that the set of actions available to the players in different states of the world is the same. Thus, the state of the world is characterized by the payoff functions of the two managers.

[8]These investment levels will in general differ from the first-best investments of the previous section.
[9]Note that the equilibrium outcomes of a mechanism are always efficient.

centives of the managers to invest for two reasons. First, ownership gives the manager partial claims to the transferable returns of the asset owned. The likelihood of different states of nature is affected by the ex ante investments of the two managers. Second, majority ownership of physical assets of firm i gives full control over action a_i. This in turn improves the bargaining position of the manager who is majority shareholder. Recall that manager may have both private and transferable payoff incentives for gaining control of action a_i ($i = 1, 2$).

6. Bargaining

Between periods t_3 and $t_3 + 1$, the owners choose actions a_1 and a_2. We model this choice of ex post actions as a bargaining game similar to that of [14][10]. The two managers M_1 and M_2 bargain over a pie of unit size. Each manager M_j ($j = 1, 2$) has an outside option b_j. Suppose that $b_1 + b_2 \leq 1$, i.e. there are always gains from trade. We also let x_j denote the equilibrium payoff to manager M_j ($j = 1, 2$). The game lasts for one bargaining period that consists of a finite number of stages $\{1, \cdots, 1/\Delta\}$ where $1/\Delta \geq 1$ is an integer. Offers follow at time intervals Δ. In each stage, nature chooses the manager that makes the offer with some probability $0 \leq \pi \leq 1$ that reflects the relative bargaining power of the two managers. The chosen manager proposes an agreement. If a responder accepts an agreement, the negotiation ends and each manager receives a payoff according to the proposed agreement. If a responder rejects the offer, the game moves to the next stage unless this is the last stage of the bargaining game. When a responder takes the outside option, the negotiation game ends and the managers receive their respective outside options.

One can show (see [14]) that the limit equilibrium payoffs (x_1, x_2) to the two managers as $\Delta \to 0$ are given by
1) If $b_1 < 1/2$ and $b_2 < 1/2$ then (x_1, x_2);
2) If $b_1 < 1/2$ then $(x_1, x_2) = (b_1, 1 - b_1)$;
3) If $b_2 < 1/2$ then $(x_1, x_2) = (1 - b_2, b_2)$.

7. Solution

<u>Case 1:</u> As a benchmark case we consider a situation where the two firms are controlled by their respective managers. That is, the majority of the shares of firm i is owned by manager M_i ($i = 1, 2$). According to the bargaining solution there are three possibilities for the outcome of the renegotiation game; neither outside option (manager's payoff if the two managers choose their

---

[10]We model the contract negotiation as a finite horizon bargaining game. This is a stylized alternating offer model developed by [15]. We use an extended version of [15] where each of the two players has an outside alternative, that is, a payoff if the parties choose actions independently.

actions independently) binds, manager M_1's outside option binds, and manager M_2's outside option binds. Thus, the set of states of nature consists of three pairwise disjoint sets

$$\Theta_0^1 = \left\{ \theta \in \Theta \middle| U_i\left(a_1^1(\theta), a_2^1(\theta), \theta \middle| s_1, s_2\right) \right.$$
$$\left. \leq \frac{1}{2} S\left(a_1^*(\theta), a_2^*(\theta), \theta\right) \text{ for } i = 1, 2 \right\} \quad (10)$$

$$\Theta_1^1 = \left\{ \theta \in \Theta \middle| U_1\left(a_1^1(\theta), a_2^1(\theta), \theta \middle| s_1, s_2\right) \right.$$
$$> \frac{1}{2} S\left(a_1^*(\theta), a_2^*(\theta), \theta\right),$$
$$U_1\left(a_1^1(\theta), a_2^1(\theta), \theta \middle| s_1, s_2\right)$$
$$\left. \leq \frac{1}{2} S\left(a_1^*(\theta), a_2^*(\theta), \theta\right) \right\} \quad (11)$$

and

$$\Theta_2^1 = \left\{ \theta \in \Theta \middle| U_1\left(a_1^1(\theta), a_2^1(\theta), \theta \middle| s_1, s_2\right) \right.$$
$$\leq \frac{1}{2} S\left(a_1^*(\theta), a_2^*(\theta), \theta\right),$$
$$U_1\left(a_1^1(\theta), a_2^1(\theta), \theta \middle| s_1, s_2\right)$$
$$\left. > \frac{1}{2} S\left(a_1^*(\theta), a_2^*(\theta), \theta\right). \right\} \quad (12)$$

The intuition of these sets of states of nature can be developed by assuming that the managers have differing private payoffs for given actions. The manager with the high private payoff will be likely to have a stronger bargaining power. This is reflected in a higher outside option.

The ex ante utilities of the two managers can be written as

$$\frac{1}{2} \sum_{\theta \in \Theta_0^1} p_\theta(e_1, e_2) S\left(a_1^*(\theta), a_2^*(\theta), \theta\right)$$
$$+ \sum_{\theta \in \Theta_1^1} p_\theta(e_1, e_2) U_1\left(a_1^1(\theta), a_2^1(\theta), \theta\right)$$
$$+ \sum_{\theta \in \Theta_2^1} p_\theta(e_1, e_2) \left(S\left(a_1^*(\theta), a_2^*(\theta), \theta\right) \right.$$
$$\left. - U_2\left(a_1^1(\theta), a_2^1(\theta), \theta\right) \right) - e_1 \quad (13)$$

This expression shows the link between the private and transferable payoffs. Each manager at the time of the investment decision places a higher weight on the states of nature that lead to higher appropriation opportunities at the bargaining stage.

$$\frac{1}{2} \sum_{\theta \in \Theta_0^1} p_\theta(e_1, e_2) S(a_1^*(\theta), \theta)$$
$$+ \sum_{\theta \in \Theta_1^1} p_\theta(e_1, e_2) \left(S\left(a_1^*(\theta), a_2^*(\theta), \theta\right) \right.$$
$$\left. - U_2\left(a_1^{12}(\theta), a_2^{12}(\theta), \theta\right) \right)$$
$$+ \sum_{\theta \in \Theta_2^1} p_\theta(e_1, e_2) U_2\left(a_1^{12}(\theta), a_2^{12}(\theta), \theta\right) - e_2 \quad (14)$$

Thus, the optimal choices of investments are determined by

$$\frac{1}{2}\sum_{\theta\in\Theta_0^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}S\big(a_1^*(\theta),a_2^*(\theta)\theta\big)$$

$$+\sum_{\theta\in\Theta_1^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}U_1\big(a_1^1(\theta),a_2^1(\theta),\theta\big) \tag{15}$$

$$+\sum_{\theta\in\Theta_2^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}\big(S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$-U_2\big(a_1^1(\theta),a_2^1(\theta),\theta\big)\big)=1$$

$$\frac{1}{2}\sum_{\theta\in\Theta_0^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$+\sum_{\theta\in\Theta_1^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}\big(S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$-U_2\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta\big)\big) \tag{16}$$

$$+\sum_{\theta\in\Theta_2^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}U_2\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta\big)=1$$

It is instructive to compare investments $\big(e_1^{SP},e_2^{SP}\big)$ with $\big(e_1^{NI},e_2^{NI}\big)$ (investments in the case of non-integration). Specifically, comparing (13) and (14) with (6), verify that $e_j^{SP}<e_j^{NI}$ for $j=1,2$. The intuition is the following. The incentives of the two parties to invest are unchanged (compared to the non-integration case) conditional on the actions being efficient. While in the case when the ex post actions are inefficient, each manager/owner internalizes only a fraction of his/her marginal returns to investment. This leads to our result on the relationship between investments.

Case 2: Manager M_1 has majority ownership of both firms. In this case, both s_1 and s_2 are strictly greater than 1/2. Thus, all of the residual property rights belong to manager M_1. This gives him decision rights over the choice of both a_1 and a_2.

Similar to the previous case we define the partition of the set of states of nature

$$\Theta_0^{12}=\Big\{\theta\in\Theta\Big|U_i\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta|s_1,s_2\big)$$

$$\le\frac{1}{2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)\quad\text{for }i=1,2\Big\} \tag{18}$$

$$\Theta_1^{12}=\Big\{\theta\in\Theta\Big|U_1\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta|s_1,s_2\big)$$

$$>\frac{1}{2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big),$$

$$U_1\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta|s_1,s_2\big) \tag{19}$$

$$\le\frac{1}{2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)\Big\}$$

$$\Theta_2^{12}=\Big\{\theta\in\Theta\Big|U_1\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta|s_1,s_2\big)$$

$$\le\frac{1}{2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big),$$

$$U_1\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta|s_1,s_2\big) \tag{20}$$

$$\le\frac{1}{2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)\Big\}$$

Accordingly the optimal investments are given by

$$\frac{1}{2}\sum_{\theta\in\Theta_0^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$+\sum_{\theta\in\Theta_1^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}U_1\big(a_1^1(\theta),a_2^1(\theta),\theta\big) \tag{21}$$

$$+\sum_{\theta\in\Theta_2^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_1}\big(S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$-U_2\big(a_1^1(\theta),a_2^1(\theta),\theta\big)\big)=1$$

$$\frac{1}{2}\sum_{\theta\in\Theta_0^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$+\sum_{\theta\in\Theta_1^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}\big(S\big(a_1^*(\theta),a_2^*(\theta),\theta\big)$$

$$-U_2\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta\big)\big) \tag{22}$$

$$+\sum_{\theta\in\Theta_2^1}\frac{\partial p_\theta(e_1,e_2)}{\partial e_2}U_2\big(a_1^{12}(\theta),a_2^{12}(\theta),\theta\big)=1$$

The important difference of this case from the previous one is that manager M_1 now has control rights over both actions which results in an increase in the set of states of nature where his outside option is binding. Thus, if these are the states that are most desirable from the efficiency point of view, then this ownership structure is optimal.

8. The Case with Two States of Nature

In this section we consider a situation where the random variable θ can assume two values, i.e. $\Theta=\{\theta_L,\theta_H\}$. Moreover, we assume that in state θ_L both transferable and private payoffs of both parties are equal to zero. One can think of a situation where the two firms are engaged in a joint research and development project, and where θ stands for the success of the venture, θ_H reflecting a situation when the research part of the project is a success. Thus, the ex ante payoff to manager $M_i(i=1,2)$ can be written as

$p(e_1,e_2)U_i\big(a_1,a_2,T,\theta_H|s_1,s_2\big)-e_i$, where we have simplified our notation by letting

$$p(e_1,e_2)\equiv p_{\theta_H}(e_1,e_2).$$

It will be important to distinguish two cases

Case 1: Strategic complements[11] $\dfrac{\partial^2 p(e_1,e_2)}{\partial e_1 \partial e_2} > 0$.

Case 2: Strategic substitutes $\dfrac{\partial^2 p(e_1,e_2)}{\partial e_1 \partial e_2} < 0$.

Under strategic complements (substitutes) the marginal benefit of manager M_i's $(i=1,2)$ investment is an increasing (decreasing) function of the other manager's investment. Note that strategic complementarity and substitutability of ex post actions is defined analogously. For example, the case of complements is easily illustrated by considering firms 1 and 2 as engaged in research and development, respectively. Clearly there are advantages to somehow coordinating these activities. For substitutes, a convenient example is actions that result in "right-sizing" of local or regional markets.

It is convenient to illustrate the alternatives in a two-way table of investments and actions (**Table 1**). Rows in this table correspond to investments and columns to actions. We also distinguish cases where both investments and actions can be strategic complements and strategic substitutes. Moreover, we examine the sensitivity of the optimal ownership shares to the importance of the manager's investment decisions. By importance we mean the relative effect of the manager's investment decision on the probability of realizing a good state of nature.

Table 1. Key parameters and optimal ownership for alternative investments and actions.

Investments \ Actions	$u_i^T(a_1,a_2,\theta_H)$ $= A_i(a_1+1)^{\alpha_i}(a_2+1)^{\beta_i}$ $u_i^P(a_1,a_2,\theta_H)$ $= (a_i+1)^{\gamma_i}$	$u_i^T(a_1,a_2,\theta_H)$ $= A_i(\sigma a_1 + a_2)^{\alpha_i}$ $u_i^P(a_1,a_2,\theta_H)$ $= (a_i+1)^{\gamma_i}$
$p(e_1,e_2)$ $= A(e_1+1)^{\delta_1}(e_2+1)^{\delta_2}$ $\delta_1=0.6; \quad \delta_2=0.2$	$s_1 \geq 0.75; s_2 \leq 0.1$	$s_1 \geq 0.75; \ s_2 \leq 0.1$
$p(e_1,e_2)$ $= A(e_1+1)^{\delta_1}(e_2+1)^{\delta_2}$ $\delta_1=0.4; \quad \delta_2=0.4$	$s_1 \geq 0.7; s_2 \leq 0.3$	$s_1 \geq 0.65; \ s_2 \leq 0.35$
$p(e_1,e_2)$ $= A(\eta e_1 + e_2)^{\delta}$ $\eta = 0.5$	$s_1 \leq 0.1; s_2 = 0$	$s_1 = 0; \ s_2 = 0$
$p(e_1,e_2)$ $= A(\eta e_1 + e_2)^{\delta}$ $\eta = 1$	$s_1 \geq 0.6; s_2 \leq 0.4$	$s_1 \geq 0.5; \ s_2 \leq 0.5$

[11]This terminology of strategic complementarity and strategic substitutability was first coined by Bulow *et al.* [16].

When investments e_1 and e_2 are strategic complements, the best response functions of the two managers. Both best response curves $BR_1(\cdot)$ and $BR_2(\cdot)$ slope upward because the marginal benefit of investment by manager M_i is increasing with e_j. Under strategic substitutability, the best-response functions are downward sloping.

Figure 2 depicts how the optimal structure is determined under strategic complementarity. Point FB in **Figure 2** corresponds to the first-best level of investments, and point NI corresponds to the investment levels chosen under non-integration. In both cases, increases in the share of either firm owned by manager M_1 result in a rightward shift in that manager's best response curve and downward shift in manager M_2's best response curve. The task of finding optimal ownership shares reduces to finding s_1 and s_2 such that the total ex ante surplus is maximized at the point of intersection of the two best-response curves.

The choice of optimal ownership structure is illustrated in **Figure 2** where we have depicted the optimal investment choices for different ownership shares. The optimal point (s_1, s_2) is given by the point of tangency between the locus of optimal investment curves and the level surface of ex ante total surplus.

It is instructive to investigate the locus LL' of optimal investments corresponding to different ownership structures. By varying either the investments or the ownership shares, which alter the slopes and intercepts of best-response curves, it is possible to trace the efficient combinations of actions and investments. Efficiency in this case means that the firms on this locus of points do not have incentives to change their investments or actions. This means that the locus of points maximizes total surplus, given ownership structure. The firms are sharing, according to the ownership structure, the maximum available surplus. We will illustrate this locus and its relationship to total surplus in the example to follow.
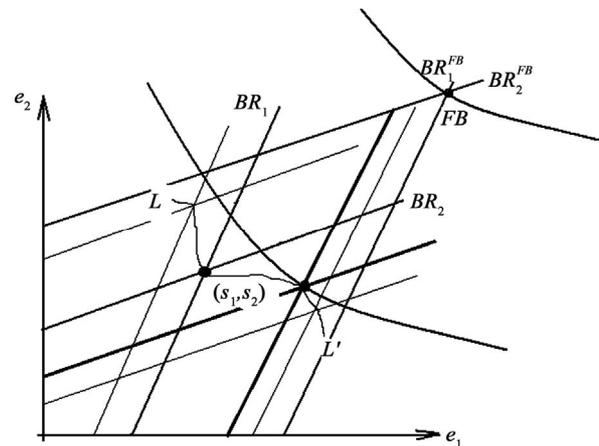

Figure 2. Optimal ownership structure.

Tables 1 and **2** contain values for the key parameters, δ_1, δ_2 and η. Additional parameters that are required to initialize the numerical example involve α_i, β_i and γ_i. These parameter choices were made to assure interior solutions for the consolidation problem. Our examination of the illustrative example suggests the problem is sensitive both to the conditioning variables and the choices of the key parameters. That is, care must be taken in the selection of parameters that there is an interior solution.

In **Table 1** the investments and the actions of manager M_1 are more important. That is the actions of manager M_1 have the larger impact on the ex post surplus. **Table 2** is similarly instructive for the situation in which manager M_2's investments and actions are the more important. If a manager is more important for both investments and actions, the optimal ownership structure is one where the majority of the shares of both firms are owned by this manager. If a manager is important for the investment but the other manager dominates the action, then the optimal ownership structure depends whether the investment or the action is the more important, has the larger effect on the ex post payoff.

In this case strategic partnerships that involve more balance in ownership of the firms are optimal.

Thus, for each specification of parameters we solve for the optimal ownership structure. The manager with important investments and actions should be a majority owner. However, the other manager may own some (possibly minority) of the shares of the two firms so that his incentives to invest are not completely muted.

Table 2. Key parameters and optimal ownership for alternative investments and actions.

Investments \ Actions	$u_i^T(a_1,a_2,\theta_H)$ $= A_i(a_1+1)^{\alpha_i}(a_2+1)^{\beta_i}$ $u_i^P(a_1,a_2,\theta_H)$ $=(a_i+1)^{\gamma_i}$	$u_i^T(a_1,a_2,\theta_H)$ $= A_i(\sigma a_1+a_2)^{\alpha_i}$ $u_i^P(a_1,a_2,\theta_H)$ $=(a_i+1)^{\gamma_i}$
$p(e_1,e_2)$ $= A(e_1+1)^{\delta_1}(e_2+1)^{\delta_2}$ $\delta_1=0.6;\ \delta_2=0.2$	$s_1 \geq 0.6;\ s_2 \leq 0.2$	$s_1 \geq 0.7;\ s_2 \leq 0.15$
$p(e_1,e_2)$ $= A(e_1+1)^{\delta_1}(e_2+1)^{\delta_2}$ $\delta_1=0.4;\ \delta_2=0.4$	$s_1 \geq 0.55;\ s_2 \leq 0.25$	$s_1 \geq 0.6;\ s_2 \leq 0.3$
$p(e_1,e_2)$ $= A(\eta e_1+e_2)^{\delta}$ $\eta=0.5$	$s_1 \leq 0.2;\ s_2 = 0$	$s_1 = 0;\ s_2 = 0$
$p(e_1,e_2)$ $= A(\eta e_1+e_2)^{\delta}$ $\eta=1$	$s_1 \geq 0.5;\ s_2 \leq 0.5$	$s_1 \geq 0.5;\ s_2 \leq 0.5$

9. Conclusions

We have examined the consolidation problem in a model which differentiates between the incentives of managers and owners. The sequence leading to a decision on the strategic partnership begins with an investment of the managers. Under circumstances that limit the possibility for complete contracts, we show the relationship between the investments of the managers, the choice of ownership structure and what we have termed the actions. In general and for the simulation there is an ownership structure that maximizes the ex ante payoff. These ownership structures are defined in terms of shares, and cover possibilities of one manager owning majority of both firms (up to complete ownership) to the two managers being majority owners of the two different firms.

The example we have carried out can be viewed as involving research as the "investment" and development as the "action". Other examples include product development as the investment and the marketing and distribution effort as the action. In short the model is sufficiently general to cover consolidation decisions in any circumstances where there is complementarity and substitutability.

The general conclusion is that for problems with this structure, the investments and the actions of the managers and the choice of ownership are highly interdependent. This implies that consolidation decisions depend on the relative importance (the affect on the transferable and non-transferable payoffs) of the relationship specific investments and ex post actions by the managers, and the uncertainty governing the outcomes as conditioned by these investments.

One of the important features of the structure of the model is the capacity that it presents for understanding the optimality of the ownership pattern. The model produces results on optimal ownership shares, i.e. shares which once taken do not provide incentives for renegotiation. The illustrative models show that there is a useful analogy between the consolidation problem, efficiency, and optimal ownership with the standard techniques of determining constrained Pareto efficient allocations. The applied implications are for a capacity to determine the ownership structure of a consolidation of two firms, under fairly general conditions. These conditions reflect the importance of the initial conditions, the relationship specific investments and actions of the managers, and the probability of success for the joint investments. The former, although not emphasized in the development of the model, are apparent from the results of the illustrative example.

In addition to these more general results on the structure of the consolidation problem and concepts of efficiency, there are specific findings that are useful in anticipating ownership structures for consolidating firms.

First, if investments are strategic complements, then separate ownership (neither firm has controlling interest, a majority of the shares, in the other) is more likely to be optimal. However, some exchange of ownership is necessary to ensure compatible incentives of the managers. Thus, for strategic complements we are more likely to observe strategic partnerships as a form of consolidation.

Second, if investments are strategic substitutes then majority ownership by one of the firms is likely to be optimal. Suppose that the relationship specific investment of manager M_1 is more important in affecting the probability of a "good" state, and suppose that the investments of both managers have a constant marginal rate of substitution. Then, it follows that firm i will be the majority owner of both firms. Thus, substitutability favors acquisition (one firm owning the majority of shares of both firms) as a form of industry consolidation.

Third, the importance of private or manager specific benefits compared to transferable benefits is a factor in determining consolidation. Managers benefit privately from their success in realizing "good" states of nature. These benefits are an integral part of consolidation decisions, given the structure of our problem and the differentiation between investments of the managers and choices of optimal ownership. If the private benefits are very important then the only channel through which ownership affects incentives to invest/take action is through the firm's acquisition of shares. Financial claims affect incentives to invest and act, but to a lesser degree.

REFERENCES

[1] S. Grossman and O. Hart, "The Costs and Benefits of Ownership: A Theory of Lateral and Vertical Integration," *Journal of Political Economy*, Vol. 94, No. 4, 1986, pp. 691-719. doi:10.1086/261404

[2] O. Hart, "Firms, Contracts, and Financial Structure," Oxford University Press, Oxford, 1995. doi:10.1093/0198288816.001.0001

[3] Y. K. Che and D. Hausch, "Cooperative Investments and the Value of Contracting: Coase vs. Williamson," *American Economic Review*, Vol. 89, No. 1, 1999, pp. 125-147. doi:10.1257/aer.89.1.125

[4] P. Aghion and J. Tirole, "On the Management of Innovation," *Quarterly Journal of Economics*, Vol. 109, No. 4, 1994, pp. 1185-1207. doi:10.2307/2118360

[5] P. Aghion and P. Bolton, "An Incomplete Contracts Approach to Financial Contracting," *Review of Economic Studies*, Vol. 59, No. 3, 1992, pp. 473-494. doi:10.2307/2297860

[6] O. Hart and J. Moore, "Incomplete Contracts and Renegotiation," *Econometrica*, Vol. 56, No. 4, 1988, pp. 755-786. doi:10.2307/1912698

[7] E. Maskin, "Nash Equilibrium and Welfare Optimality," *Review of Economic Studies*, Vol. 66, No. 1, 1999, pp. 23-36. doi:10.1111/1467-937X.00076

[8] J. Moore and R. Repullo, "Subgame Perfect Implementation," *Econometrica*, Vol. 56, No. 5, 1998, pp. 1191-1220. doi:10.2307/1911364

[9] T. Palfrey, "Implementation Theory," Forthcoming in R. Aumann and S. Hart, Eds., *Handbook of Game Theory*, Vol. 3, North-Holland, Amsterdam, 1998.

[10] E. Maskin and J. Moore, "Implementation and Renegotiation," *Review of Economic Studies*, Vol. 66, No. 1, 1999, pp. 39-56. doi:10.1111/1467-937X.00077

[11] I. Segal, "Complexity and Renegotiation: A Foundation for Incomplete Contracts," *Review of Economic Studies*, Vol. 66, No. 1, 1999, pp. 57-82. doi:10.1111/1467-937X.00078

[12] E. Maskin and J. Tirole, "Unforeseen Contingencies, and Incomplete Contracts," *Review of Economic Studies*, Vol. 66, No. 1, 1999, 83-114. doi:10.1111/1467-937X.00079

[13] J. Tirole, "Incomplete Contracts: Where Do We Stand?" *Econometrica*, Vol. 67, No. 4, 1999, pp. 741-781. doi:10.1111/1468-0262.00052

[14] W. B. MacLeod and J. M. Malcomson, "Investments, Holdup, and the Form of Market Contracts," *American Economics Review*, Vol. 83, No. 4, 1993, pp. 811-837.

[15] A. Rubinstein, "Perfect equilibrium in a bargaining model," *Econometrica: Journal of the Econometric Society*, Vol. 50, No. 1, 1982, 97-109. doi:10.2307/1912531

[16] J. Bulow, J. Geanakoplos and P. Klempere, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, Vol. 93, No. 3, 1985, pp. 488-511. doi:10.1086/261312