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Natural Convective Mass Transfer MHD Flow of Chemically Reactive Micropolar Fluid Past a Vertical Porous Plate

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Authors' contributions

This work was carried out in collaboration between all authors. Author KEH designed the study, noted the protocol and wrote the first draft of the manuscript. Author SMMH managed the literature searches and developed the flow model. Author SAR analyzed the numerical process. Author MMH provided overall guidance and assisted in interpretation of the findings. All authors read and approved the final manuscript.

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ABSTRACT

Magnetic field effects on a free convective mass transfer flow of chemically reactive micropolar fluid over a vertical porous plate are investigated in this work. Suitable similarity transformations are used to derive a set of nonlinear ordinary differential equations governing the flow. Analytical solution of the dimensionless problem is obtained using perturbation technique. The velocity, angular velocity and concentration profiles, which are controlled by a number of parameters, are presented graphically. Based on these graphs the conclusions are depicted, and the obtained results are tested for their accuracy.

Keywords: MHD; micropolar fluid; porous plate; chemical reaction; perturbation technique.

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NOMENCLATURES

u, v	:	Velocity component
<i>g</i>	:	Acceleration
$\pmb{\beta}^*$:	concentration expansion coefficient
B_0	:	Magnetic field component
χ	:	Vortex viscosity
γ	:	Spin gradient viscosity
υ	:	Viscosity
ho	:	Density
S_{c}	:	Schmidt number
D_m	:	Diffusion coefficient
C_r	:	Chemical reaction parameter
C_{∞}	:	Concentration of uniform flow
Δ	:	Micro-rotational number
G_m	:	Modified Grash of number
K	:	Permeability of porous palte
М	:	Magnetic force number
\wedge	:	Spin gradient viscosity number
λ	:	Vortex viscosity

1. INTRODUCTION

In recent years, the theory of micropolar fluid has fascinated momentous attention among the researchers. Indeed, the theory of micropolar fluids is introduced by Eringen [1] can be used to explicate the flow characteristics of polymers, paints, animal blood, lubricants, liquid crystals, hematological suspensions, geomorphological sediments and geomorphological sediments etc. The theory described about local effect increasing due to microstructure and the intrinsic motion of the fluid elements. The model of micropolar fluid represents fluids consisting of rigid, randomly generated particles suspended in a viscous medium where the deformation of the particles is neglected. Kucaba-Pietal [2], Khedr et al. [3] investigated colloidal, Muthu et al. [4] analvzed human and animal blood, Lockwood et al. [5] driven liquid crystal as well as exotic lubricants. Kelson and Desseaux [6] investigated the effect of surface circumstance on micropolar fluid flow. The unsteady micropolar fluid flow between two parallel porous plates was driven Srinivasacharya et al. [7]. Mixed convection micropolar fluid on a porous stretching sheet is investigated by Bhargava et al. [8]. Tripathy et al. [9] has investigated hydromagnetic micropolar fluid on a porous stretching sheet. Mohanty et al. [10] investigated heat and mass transfer effect on micropolar fluid on a porous stretching sheet. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media has been studied by Lukaszewicz [11].

Fluid dynamics is the area of science which is related with the study of motion of fluids. Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in the presence of a magnetic field. The study of Magnetohydrodynamics has numerous applications such as induction flow meter, cooling of nuclear reactors, structure of stars and planets. Recently, the application of Magnetohydrodynamics in the polymer industry and metallurgy has fascinated the attention of many scholars.

Free convection flow takes place frequently in nature, flows of fluid through porous media are of significant interest now days and have fascinated by many researchers owing to their applications in the science and technology. Study of fluid flow in porous medium is based upon the empirically Darcy's law. Such flows are assumed to be useful in diminishing the free convection.

Along with the free convection flow, the phenomenon of mass transfer is also very significant in the theories of stellar structure. The study of convective flow with mass transfer along a vertical porous plate is receiving paramount interest of many scholars because of its numerous applications in the field of geophysical and cosmical sciences. Prathap Kumer et al. [12] studied on free convection flow of micropolar and viscous fluids through a vertical duct. Raptis [13], Samiulhaq et al. [14] and Seth et al. [15] studied on free convective oscillatory flow and mass transfer with ramped temperature on a porous plate. Thereafter, Chamkha et al. [16], Chaudhary et al. [17], Samad and Mohebujjaman [18], Eldabe et al. [19] and Seth et al. [15] have paid attention to the study of MHD free convection and mass transfer flows.

The chemical reaction rate counts on the concentration of the species itself. In many chemical engineering systems, there is the chemical reaction between a foreign mass and the fluid. This system occurs in many industrial applications such aspolymer production, manufacturing of ceramics and food processing. Mishra et al. [20] has analyzed the effect of chemical reaction on hydromagnetic micropolar fluid flow. Raju et al. [21] investigated an unsteady free convection and chemically

reactive MHD flow through infinite vertical porous plate. Bakr [22] has driven the effect of chemical reaction on micropolar fluid with oscillatory plate. Das et al. [23] analyzed the effective of first order homogeneous chemical reaction of an unsteady micropolar fluid flow. Ibrahim et al. [24], Anand Rao et al. [25], Das [26] and Raju et al. [21] investigated the effect of chemical reaction on an unsteady MHD free convection fluid through semi-infinite vertical porous plate with heat absorption. Bakr [22] analyzed the characteristic of a micropolar fluid velocity on oscillatory plate and constant heat source in a rotating frame. Kucaba-Pietal [2], Khedr et al. [3] investigated the micro inertia effects on the flow of a micropolar fluid past a semi-infinite plate.

Hence our main goal is to investigate a free convective mass transfer steady flow of a chemically reactive micropolar fluid past a semiinfinite porous plate.

2. FORMULATION OF THE PROBLEM

A natural convective mass transfer steady flow of a chemically reactive micropolar fluid through a semi-infinite vertical porous plate is taken into account at the presence of magnetic field. The flow is considered vertically by x-direction and ydirection is represented horizontally. When the flow at rest, the species concentration level $C = C_{\infty}$ at all point, where C_{∞} be the concentration of uniform flow. It is also assumed that a magnetic field *B* of uniform strength is applied normal to the flow region. The physical configuration and co-ordinate system of the problem is presented in the following Fig. 1.

Within the framework of the above stated assumption, the governing equations under the boundary-layer approximations are given by,

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta^* \left(C - C_{\infty}\right) + \left(v + \frac{\chi}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y} - \frac{\sigma' uB_0^2}{\rho} - \frac{v}{K_1}u$$
(2)

Angular Momentum Equation

$$u\frac{\partial\Gamma}{\partial x} + v\frac{\partial\Gamma}{\partial y} = \frac{\gamma}{\rho j} \left(\frac{\partial^2\Gamma}{\partial y^2}\right) - \frac{\chi}{\rho j} \left(2\Gamma + \frac{\partial u}{\partial y}\right)$$
(3)

Concentration Equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_c \left(C - C_\infty\right)$$
(4)

with boundary condition,

$$u = 0, \ \Gamma = -s \frac{\partial u}{\partial y}, \ C = C_w \text{ at } y = 0$$

 $u = 0, \ \Gamma = 0, \quad C = C_{\infty} \text{ at } y \to \infty$

where u is the velocity component, Γ is the velocity acting in z - direction (the rotation of Γ is in the x-y plane), B_0 is the magnetic field component, g is local acceleration due to gravity, χ is the vortex viscosity, γ is the spin gradient viscosity, β^* is concentration expansion coefficient.



Fig. 1. Physical configuration of the flow

2.1 Non-dimensional Form

Since our goal is to attain analytical solutions of the problem so we introduce the following dimensionless variables,

$$\eta = y \sqrt{\frac{U_0}{2vx}} , \quad \psi = \sqrt{2vU_0 x} f(\eta) , \quad u = \frac{\partial \psi}{\partial y} \text{ and}$$
$$v = -\frac{\partial \psi}{\partial x}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}} \text{ and } \Gamma = \sqrt{\frac{U_0^3}{2vx}} g(\eta)$$

The dimensionless equations are,

$$(1+\Delta) f'''(\eta) + \Delta g'(\eta) + f(\eta) f''(\eta) + G_{_M} \varphi(\eta) - (K+M) f'(\eta) = 0$$
(5)

$$\wedge g''(\eta) + f'(\eta)g(\eta) + f(\eta)g'(\eta) - 2\lambda g(\eta) - \lambda f''(\eta) = 0$$
(6)

$$\varphi''(\eta) + S_c f(\eta) \varphi'(\eta) - S_c C_r \varphi(\eta) = 0$$
⁽⁷⁾

The associate boundary conditions

$$f(\eta) = f_w, \qquad f'(\eta) = 0, \qquad g(\eta) = -sf''(\eta),$$

$$\varphi(\eta) = 1 \quad \eta = 0$$

$$f'(\eta) = 0, \ g(\eta) = 0, \ \varphi(\eta) = 0 \ \eta \to \infty$$

Where, micro-rotational number $\Delta = \frac{\chi}{\rho v}$, modified Grashof number, $G_m = \frac{g\beta^*(C_w - C_w)2x}{U_0^2}$ permeability of porous plate, $K = \frac{2vx}{K_1U_0}$, magnetic force number, $M = \frac{\sigma B_0^2 2x}{U_0 \rho}$, Spin Gradient number, $\Lambda = \frac{\gamma}{v \rho j}$, Vortex viscosity, $\lambda = \frac{2x\chi}{\rho j U_0}$, Schmidt number, $S_c = \frac{v}{D_m}$, chemical reaction parameter, $C_r = K_c \frac{2x}{U_0}$.

3. ANALITICAL SOLUTION

Since the solution is sought for the large suction further transformation can be made Arifuzzaman et al. [27] as,

$$\xi = \eta f_{w} \tag{7}$$

$$f(\eta) = f_w F(\xi) \tag{8}$$

$$\phi(\eta) = f_w^2 G(\xi) \tag{9}$$

$$g(\eta) = f_w^{3} H(\xi)$$
(10)

Now the model with small quantity,

$$(1+\Delta)F'''+\Delta H'+FF''+\varepsilon G_MG-(K+M)\varepsilon F'=0$$
(11)

$$\wedge H'' + F'H + FH' - 2\lambda \mathcal{E}H - \lambda \mathcal{E}F'' = 0 \qquad (12)$$

$$G'' + S_c G'F - \varepsilon S_c C_r G = 0 \tag{13}$$

The associate boundary conditions

$$F(\xi) = 1, \qquad F'(\xi) = 0, \qquad G(\xi) = \varepsilon,$$

$$H(\xi) = -sF''(\xi) \text{ at } \xi = 0$$

$$F'(\xi) = 0, \ G(\xi) = 0, \ H(\xi) = 0 \ \text{at} \ \xi \to \infty$$

Now for the large suction $(f_w > 1)$, \mathcal{E} will be very small. Therefore following F, G and H can be expended in terms of the small perturbation quantity ε ,

$$F(\xi) = 1 + \varepsilon F_1(\xi) + \varepsilon^2 F_2(\xi) + \varepsilon^3 F_3(\xi) + \dots \dots$$
(14)

$$G(\xi) = \varepsilon G_1(\xi) + \varepsilon^2 G_2(\xi) + \varepsilon^3 G_3(\xi) + \dots$$
(15)

$$H(\xi) = \varepsilon H_1(\xi) + \varepsilon^2 H_2(\xi) + \varepsilon^3 H_3(\xi) + \dots$$
(16)

The dimensionless equations (5) - (7) transform into the following first order, second order and third order equations with their associated boundary conditions,

$$(1+\Delta) F_{1}^{'''} + \Delta H_{1}' + F_{1}^{''} = 0$$

$$\wedge H_{1}'' + H_{1}' = 0$$

$$G_{1}'' + S_{c}G_{1}' = 0$$
(17)

$$F_1 = 0, \ G_1 = 1, \ H_1 = 0 \ \text{at} \ \xi = 0$$

$$F_1'=0, \ G_1=0, \ H_1=0 \ \text{at} \ \xi \to \infty$$

$$(1+\Delta)F_{2}^{'''}+\Delta H_{2}'(\xi)+F_{1}F_{1}''+F_{2}''+G_{M}G_{1}(\xi)-(K+M)F_{1}'=0$$

$$\wedge H_{2}''+F_{1}'H_{1}+F_{1}H_{1}'+H_{2}'-2\lambda H_{1}-\lambda F_{1}''=0$$

$$G_{2}''+S_{c}G_{2}'+S_{c}G_{1}'F_{1}-S_{c}C_{r}G_{1}=0$$
(18)

$$F_2 = 0, \qquad G_2 = 0, \quad H_2 = 0 \text{ at } \xi = 0$$

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$$F_2' = -\frac{1}{\varepsilon^2}, \ G_2 = 0, \ H_2 = 0 \text{ at } \xi \to \infty$$

 $(1+\Delta)F_{3}'''+\Delta H_{3}'+F_{2}F_{1}''+F_{1}F_{2}''+F_{3}''+G_{M}G_{2}-(K+M)F_{2}'=0$ $\wedge H_{3}''+F_{2}'H_{1}+F_{1}'H_{2}+F_{2}H_{1}'+F_{1}H_{2}'+H_{3}'-2\lambda H_{2}-\lambda F_{2}''=0$ $G_{3}''+S_{c}G_{3}'+S_{c}G_{2}'F_{1}+S_{c}G_{1}'F_{2}-S_{c}C_{r}G_{2}=0$ (19)

 $F_1 = 0, \ G_1 = e^{-S_c \xi}, \ H_1 = 0$

$$F_3 = 0, \ G_3 = 0, \ H_3 = 0 \text{ at } \xi = 0$$

 $F_3' = \frac{1}{\epsilon^3}, \ G_3 = 0, \ H_3 = 0 \text{ at } \xi \to \infty$

Now the solution of first order, second order and third order equations are given following,

$$F_{2} = \frac{1}{A_{2}\varepsilon^{2}} - \frac{\xi}{\varepsilon^{2}} - \frac{1}{A_{2}\varepsilon^{2}}e^{-A_{2}\xi} + A_{3}e^{-S_{c}\xi}, H_{2} = 0 \text{ and } G_{2} = -C_{r}e^{-S_{c}\xi}$$

$$F_{3} = -\frac{1}{A_{2}\varepsilon^{3}} + \frac{\xi}{\varepsilon^{3}} + \frac{1}{A_{2}\varepsilon^{3}}e^{-A_{2}\xi} + A_{15}e^{-A_{2}\xi} + A_{16}e^{-S_{c}\xi} + A_{17}e^{-S_{c}\xi} + A_{18}e^{-A_{2}\xi} - A_{19}e^{-S_{c}\xi}$$

$$H_{3} = A_{8}e^{-A_{2}\xi} + A_{9}e^{-S_{c}\xi}, G_{3} = A_{4}e^{-S_{c}\xi} - A_{5}e^{-S_{c}\xi} + A_{6}\xi e^{-S_{c}\xi} - A_{7}e^{-(S_{c}+A_{2})\xi} + A_{20}e^{-2S_{c}\xi}$$

$$(20)$$

Substituting the values of F, H and G, We get

The velocity equation,

$$f = f_{w}F(\xi)$$

$$\Rightarrow f = f_{w} + \left(\varepsilon^{2} f_{w}A_{3} + \varepsilon^{3} f_{w}A_{16} + \varepsilon^{3} f_{w}A_{17}e^{-S_{c}\xi} - \varepsilon^{3} f_{w}A_{19}\right)e^{-S_{c}\xi} + \left(\varepsilon^{3} f_{w}A_{15} + \varepsilon^{3} f_{w}A_{18}\right)e^{-A_{2}\xi}$$

$$\Rightarrow f' = \left(-S_{c}\varepsilon^{2} f_{w}A_{3} - S_{c}\varepsilon^{3} f_{w}A_{16} - S_{c}\varepsilon^{3} f_{w}A_{17}e^{-S_{c}\xi} + S_{c}\varepsilon^{3} f_{w}A_{19}\right)e^{-S_{c}\xi}$$

$$-\left(A_{2}\varepsilon^{3} f_{w}A_{15} + A_{2}\varepsilon^{3} f_{w}A_{18}\right)e^{-A_{2}\xi}$$
(21)

The angular velocity equation,

$$g(\eta) = f_w^3 H(\xi)$$

$$\Rightarrow g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2 \xi} + \varepsilon^3 f_w^3 A_9 e^{-S_c \xi}$$
(22)

The concentration equation,

$$\phi(\eta) = f_{w}^{2}G(\xi)$$

$$\Rightarrow \phi(\eta) = f_{w}^{2}\varepsilon e^{-S_{c}\xi} - \varepsilon^{2}f_{w}^{2}C_{r}e^{-S_{c}\xi} + \varepsilon^{3}f_{w}^{2}A_{4}e^{-S_{c}\xi} - \varepsilon^{3}f_{w}^{2}A_{5}e^{-S_{c}\xi} + \varepsilon^{3}f_{w}^{2}A_{6}\xi e^{-S_{c}\xi}$$

$$-\varepsilon^{3}f_{w}^{2}A_{7}e^{-(S_{c}+A_{2})\xi}$$
(23)

4. RESULTS AND DISCUSSION

For the purpose of the applicability of the present mathematical model, the analytical solution are driven using the perturbation method and the discussion is made for various values of parameters just as Modified grashof number G_m , Suction parameter f_w , Magnetic force number M, permeability of porous plate K, Micro-rotational number Δ , Vortex viscosity λ , Spin gradient viscosity number (\wedge) , Schmidt number S_c and Chemical reaction parameter C_r . The fluid velocity, angular velocity and concentration versus the non-dimensional coordinate variable η are displayed in Figs. 2 - 4.



Fig. 2. Velocity profiles for different values of (a) modified Grashof number (b) suction parameter (c) magnetic force number (d) permeability of porous plate (e) spin gradient viscosity number



Fig. 3. Angular velocity profiles for different values of (a) modified Grashof number (b) Schmidt number (c) suction parameter (d) micro-rotational number (e) spin gradient viscosity number (f) vortex viscosity



Fig. 4. Concentration profiles for different values of (a) suction parameter (d) Schmidt number (c) micro-rotational number (d) chemical reaction parameter

The velocity profiles are illustrated in Fig. 2. The increase values of magnetic parameter create a drag force known as Lorentz force. As it is observed, the velocity profiles curve climb up at the increase of magnetic force number. Modified Grashof number signifies the effect of buoyancy force to the viscous hydrodynamic force. The effect of buoyancy force, velocity curves increase with modified Grashof number (G_m) . It is observed that, the effect of permeability parameter (k) at the present of porous medium causes higher restriction to the flow. Due to increase of permeability parameter, the velocity decrease which profiles agrees with Krishnamurthy et al. [28]. Afterwards, the suction parameter (f_w) stabilizes the boundary layer growth. So the velocities profiles curve decline with go up suction parameters. Schmidt number decreases the molecular diffusivity. Then with increase of vortex viscosity, the velocity profiles plunge.

The effect of different physical parameters on angular velocity is demonstrated in Fig. 3. The microrotational effect distinguishes the micropolar fluid. It is notice that, the Modified Grashof number and micro-rotational number enhance angular velocity profiles. The effect of Schmidt number is not remarkable but suction parameter, spian gradient number, vortex viscosity decrease angular velocity profiles. The presence of porous matrix, vortex viscosity decrease angular velocity which agrees with Mishra et al. [20]. Fig. 4 describes the concentration profiles. It is observed that concentration profiles affected by chemical reaction which is good agreement with the previous result of Krishnamurthy et al. [28], Tripathy et al. [9]. The chemical reaction parameter declines the concentration of the spices in the boundary layer. The concentration profile decreases for Schmidt number which agrees with Mishra et al. [20]. Concentration profiles calculated for higher values of Suction parameter where concentration is more decrease for $f_w = 29.0$ rather than $f_w = 25.0$. The effect of micro-rotational parameter on concentration profile is not remarkable.

5. CONCLUSIONS

Some of the important findings of the present work obtained from the graphical representation of the results are listed below:

- 1. The fluid velocity and angular velocity profile decreases with the increase of Modified Grashof number.
- 2. The velocity and angular velocity profile decreases with the increase of Suction parameter and also the concentration profile decreases with the increase of Suction parameter.
- 3. The velocity profile decreases and angular velocity profiles decreases with the increase of Schmidt number and also the concentration profile decreases with the increase of Schmidt number.
- 4. The velocity profile increase with Magnetic force number.
- 5. The velocity profiles decreases with the increase of Permeability of porous plate.
- 6. The concentration profile decreases with the increase of Chemical reaction parameter.
- The angular velocity profile increases with the increase of Micro-rotational number and the concentration profile decreases with the increase of Micro-rotational number.
- 8. The angular velocity profile decreases with the increase of Spin gradient viscosity number.
- 9. The angular velocity profile decreases with the increase of Vortex viscosity.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

The expression of the coefficients A_1 , A_2 , A_3 , A_5 etc. involved in (20), (21), (22) and (23) should be read as

$A_1 = \frac{1}{\wedge},$	$A_2 = \frac{1}{1 + \Delta},$
$A_{3} = \frac{-A_{2}G_{M}}{\left(-S_{c}^{3} + A_{2}S_{c}^{2}\right)},$	$A_5 = \frac{S_c}{A_2 \varepsilon^2},$
$A_6 = \frac{S_c}{\varepsilon^2}$	$A_{7} = \frac{S_{c}^{2}}{A_{2}\varepsilon^{2} \left(A_{2}^{2} + S_{c}A_{2}\right)},$
$A_8 = \frac{-\frac{A_2\lambda}{\varepsilon^2}}{\left(A_2^2 - A_1A_2\right)},$	$A_{9} = \frac{A_{3}\lambda S_{c}^{2}}{S_{c}^{2} - A_{1}S_{c}},$
$A_{10} = \Delta A_2^2 A_8$	$A_{11} = \Delta A_2 A_9 S_c,$
$A_{12} = G_m C_r A_2,$	$A_{13} = \frac{A_2(K+M)}{\varepsilon^2},$
$A_{14} = A_2 S_c A_3 (K + M),$	$A_{15} = \frac{A_{10}}{A_2^2}$
$A_{16} = \frac{A_{11}}{\left(-S_c^3 + S_c^2 A_2\right)},$	$A_{17} = \frac{A_{12}}{\left(-S_c^3 + S_c^2 A_2\right)},$
$A_{18} = \frac{A_{13}}{A_2^2},$	$A_{19} = \frac{A_{14}}{\left(-S_c^3 + S_c^2 A_2\right)}$
$A_{20} = \frac{A_3}{2}$	

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