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Evolution Model Analysis of Group Buying Games

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This paper introduces an evolutionary game model in a mixed population. The simulation results show that competition between suppliers increases social welfare and can reveals the base price of a product in cases of some strategy profiles; however, collusion will benefit suppliers. The bounded rationality of individuals has important influence on the coexistence of suppliers.

Keywords: Group buying; evolutionary games; bounded rationality.

1. INTRODUCTION

In recent years, the phenomenon of group buying games attracted much attention. Anand and Aron established a model to show that group buying can be a price discovery mechanism in an uncertain market [1]. Chen et al. [2] analyzed the performance of a group buying auction model. Chen and Roma considered the competitive retailers' choice of group buying under given quantity discount schedules [3]; and find that symmetric competitive retailers always have higher profits based on group buying, and the supplier also has a chance to be good.

Under the research of price discounts by Dolan [4], in the paper [5], Schotanus proposed a price discount model in group buying according to the practical data. In particular, GB suppliers provide group buying with a quantity discount such that

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$w(q) = a + \frac{d}{q^e}$, $de > 0$, where q is the purchase quantity, a denotes the base wholesale price, the theoretical minimum (maximum) wholesale price; $|d|$ represents the discount level; e reflects how quickly the wholesale price decreases with the purchase quantity. It shows that this general discount schedule fits well for 66 discount schedules found in practice, with e varying from -1.00 to 1.60 in [5]. And this discount schedule was widely used, e.g. [6-10]. Except for price discounts proposed by suppliers, in games, decisions of buyers (individuals) are also important. There are many mutual learning rules in games, for example, learning from specific individuals, deciding by approximate best response or best response etc., [11,12]. These results provide a basis for the further exploration of the group-buying price discount and individuals' decision in this paper.

Based on above studies, in consideration of the competition between suppliers and the learning of individuals, this paper sets two kinds of suppliers—group buying (GB) and individual purchasing (IP); to maximize profit in the process of games, they have two strategies: scale priority and price priority strategy. Individuals also have two strategies: learn from individuals with public information in groups or react with best response. From the simulation to analyze the evolution of suppliers' strategies, the behavior of individuals, price discovery and social welfare.

2. EVOLUTIONARY GAMES MODEL

We assume that a population has N individuals. Considering the competition of suppliers, this assumes that there are two types of suppliers to provide a kind of product: (1) providing individual purchasing(IP); the unit price of a product is determined by $w_1 = a + d_1$; (2) providing group buying(GB); the unit price of a product is determined by $w_2 = a + \frac{d_2}{q^e}$, $d_2 > 0$, $e > 0$, which is the same as the discount schedule in [5], where q is the number of individuals choosing GB;. The profits of GB and IP suppliers are $U_{GB} = w_2 q$ and $U_{IP} = w_1(N - q)$, respectively. To achieve maximum profit, GB supplier (or IP supplier) can choose one of the following two strategies:

(1) If sales volume increase in the current round of game, the supplier increases d_1 (or d_2) in the next round; in the case of sales volume decreasing, the supplier reduces d_1 （or d_2) to attract more consumers;
(2) If sales volume increase in the current round of game, the supplier remains d_1 (or d_2); in the case of sales volume decreasing, the supplier reduces d_1 (or d_2); Thus, for the two strategies of suppliers, the first one prefers the price to sales volume, we called it price priority strategy S_p ; the second one pays more attention to the expansion of sales volume, we called it scale priority strategy S_s .

For individuals in the population, an individual i can choose GB or IP in t-th round, the strategy of individual i is denoted as $S(i,t)=S_{GB}(i,t)$ if it choose GB; similarly, $S_{IP}(i,t)$ represents that i choose IP. We assume that a small part of individuals' information is public in the population (their choice of strategies are public).

To achieve maximum profit (that is minimum cost for buying a product), an individual i make the decision by one of the following two rules:

(a) Learn from the individuals which have public information. Unit product cost for an individual i is denoted $U(i,t)$, in t-th round, by choosing a random j with public information, if $U(j,t) \geq U(i,t)$, this i remains strategy $S(i,t)$ unchanged in the next round, that is, $S(i,t+1) = S(i,t)$; if $U(j,t) < U(i,t)$ and $S(j,t) \neq S(i,t)$, individual i adopts j 's strategy with probability p_{ij} in the $(t+1)$ round, where

$$p_{ij} = \frac{1}{1 + \exp(-\frac{U(i,t) - U(j,t)}{k})}, k > 0 .$$

As $k \to \infty$, we have $p_{ij} \to \frac{1}{2}$, that is, i makes decision randomly between $S(i,t)$ and $S(j,t)$; while $k \to 0$ results in $p_{ij} \to 1$.

(b) Decide on best response. If the number of choosing GB is q in t-th round, an individual i makes the best response as follows: if i's strategy is $S_{IP}(i,t)$ in t-th round, then

$$S(i,t+1)=\begin{cases} S_{GB}(i,t+1), a+\frac{d_2}{(q+1)^e} < a+d_1 \\ S_{IP}(i,t+1), a+\frac{d_2}{(q+1)^e} \geq a+d_1 \end{cases};$$

similarly, if individual i's strategy is $S_{GB}(i,t)$, then

$$S(i,t+1)=\begin{cases} S_{GB}(i,t+1), a+\frac{d_2}{q^e} < a+d_1 \\ S_{IP}(i,t+1), a+\frac{d_2}{q^e} \geq a+d_1 \end{cases}$$

In the population, we assume that there is a certain preference for each individual choosing (a) or (b), an individual i choose learning from the individual with the public information is denoted as $o(i)=L$; similarly, $o(i)=B$ means that individual i choose the best response strategy.

3. EVOLUTION SIMULATION ANALYSIS

We set the size N of a population with $N=1000$. Monte Carlo simulation analysis is carried out on the game model in section 2. Suppose there are two suppliers, one is GB supplier, another is IP supplier, and both of them have the same base price a with $a=1$. And they have the same discount price in the first round such that $d_1=d_2=1$. In the case of other parameters are determined ($0<e<1, 0<k<2$), the simulation results show that, the change of the values of e and k will affect the convergence speed of evolution, but it does not affect the convergent result; in the following simulation, we take constant values for e and k with $e=0.5$ and $k=1$.

Assuming that the frequency of individual choosing GB in the population is $x_{GB}(t)$ in t-th round, that is $x_{GB}(t) = \left|\{i: S_{GB}(i,t)\}\right|/N$. The frequency of learning from the individuals which have public information is y_0, that is $y_0=\left|\{i: o(i)=L\}\right|/N$.

(I) Both suppliers choose the strategy S_s; specifically, when the profit in t-th round is lower than that in (t-1)-th round, they select $d_1(t+1)$ and $d_2(t+1)$ with $d_1(t+1)=0.9d_1(t)$, $d_2(t+1)=0.9d_2(t)$. Do 50 times random experiment with $x_{GB}(1)=0.1$ and $y_0=0.9$ as initial values, each iterates (evolution) 100 rounds. Evolution results show that, the frequency of strategy, individuals' cost and supplier's profit, all tend to be stable. As shown in Fig. 1, it is the result of one of random experiment. Coexistence of two types of suppliers are shown, and all indexes have reached a steady state (horizontal axis is of iteration times or transaction times).

As shown in Fig. 2, the horizontal axis is of random experiment times, each after 100 rounds of evolution; the frequency of all individuals choosing GB is 60%; the cases of coexistence of GB and IP is 40%; the GB supplier share most of profits. In all cases, average cost of a product converges to 1; the competition between suppliers makes $d_1(t), d_2(t)$ from $d_1(1)=d_2(1)=1$ to $d_1(100)$, $d_1(100)<0.5$ in many cases, and this lead to the unit product's price w_1 and w_2 close to the base price a; we can find that the cases with $d_2(100)=1$ correspond to the GB supplier monopolizes the whole market, and the GB supplier's scale expansion has remarkable effect. However, individuals' average cost decreased significantly, and the competition increases the social welfare.

The simulation shows that with the increasing of $x_{GB}(1)$, the coexistence of GB and IP individuals will reduce at steady state; we can find that when $x_{GB}(1) \geq 0.32$, IP individuals will disappear; similarly, with the increasing of y_0, the cases of coexistence will increase; when $y_0 < 0.65$, there is no IP individuals; this shows that the preference of decision in some range has important influence to the diversity of supplier's types.

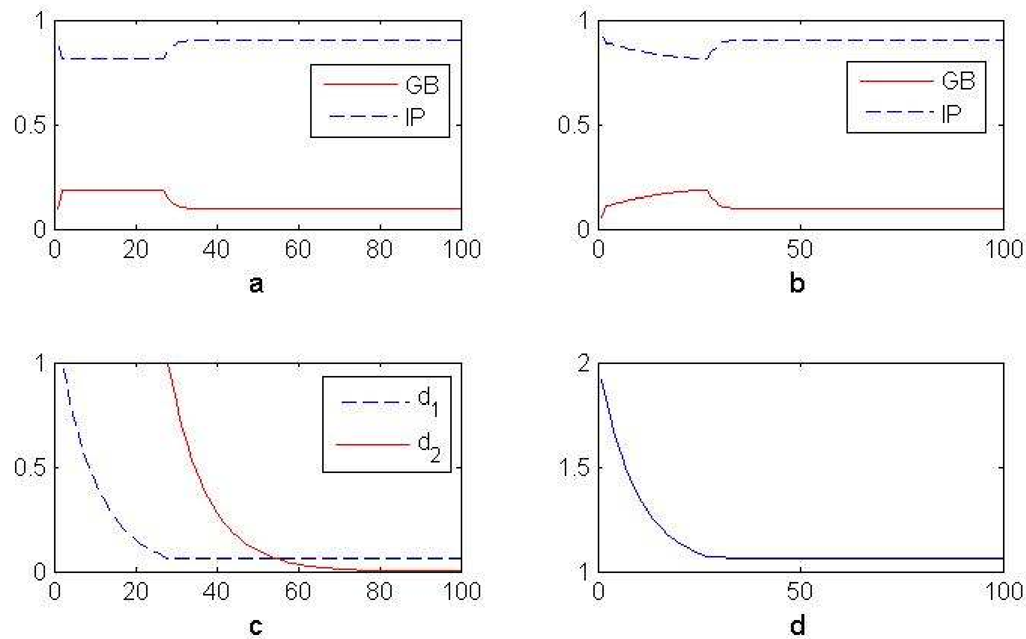

Fig. 1. a: The frequency of individuals; b: The percentage of suppliers' profit; c: The values of d_1, d_2; d: Individuals' average cost for a product

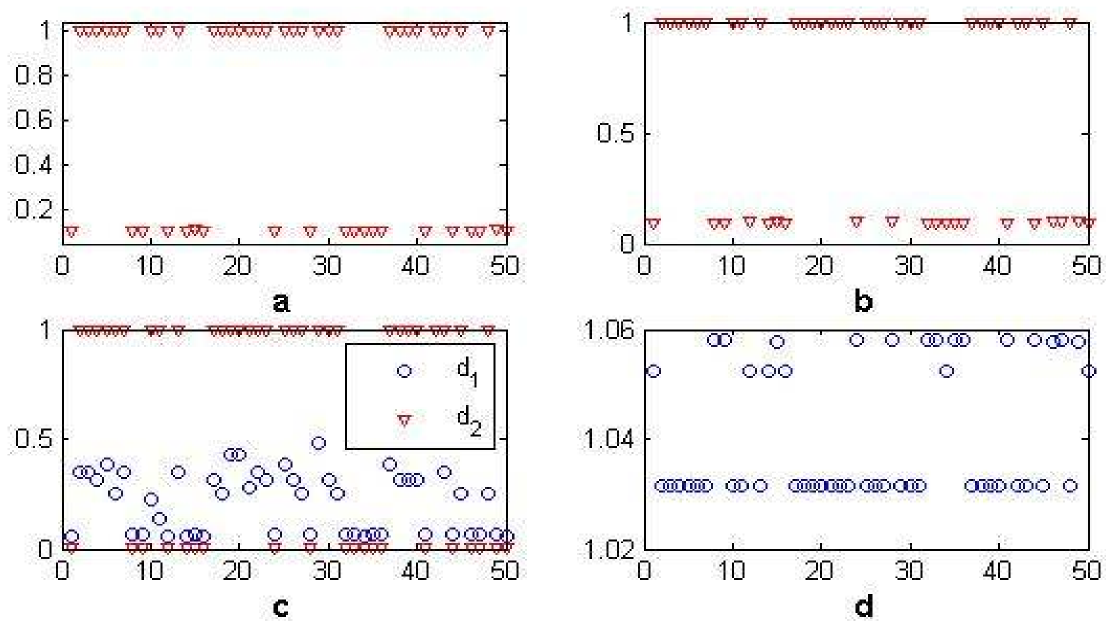

Fig. 2. a: The frequency of GB individuals; b: The percentage of the GB supplier' profit; c: The values of d_1 and d_2; d: Individuals' average cost for a unit product

Next, we consider there are differences in discount amplitude between the GB and IP supplier. Let $x_{GB}(1)=0.1$, $y_0=0.9$, if $d_1(t)=0.9d_1(t-1)$, $d_2(t)=0.5d_2(t-1)$, the result is shown as Fig. 3, it can be observed that in most of cases, GB and IP coexist, the frequency of IP is also enhanced; with the fierce competition between suppliers, this leads to the fact that $d_1(\mathrm{t}), d_2(t)\to 0$.

(Ⅱ) Both suppliers select the strategy S_p ; specifically, when the profits of the suppliers in t -th round is higher than that in (t -1)-th round, they select $d_1(t+1)$ and $d_2(t+1)$ with $d_1(t+1)=1.1d_1(t)$, $d_2(t+1)=1.1d_2(t)$, otherwise $d_1(t+1)=0.9d_1(t)$, $d_2(t+1)=0.9d_2(t)$. Let $d_1(t),\ \ d_2(t)\le 5$ (assume the price arrange for a product is in an interval [1, 6]). Run 50 times random experiment with $x_{GB}(1)=0.1$ and $y_0=0.9$ as initial values, each converges to a steady state after iterating (evolution) 100 rounds.

The result is shown in Fig. 4, two suppliers coexist in all cases; individuals' average cost near the peak value in many cases; suppliers' profits, like a result of conspire, are better than (I). Using the method in (I) (fixed one of x_{GB} and y_0, make the other changes), it can also find the thresholds of parameters which make IP individuals disappear in steady states.

(Ⅲ) The GB supplier selects the strategy S_p; as its profit in t -th round is better than that in (t -1)-th round, let $d_2(t+1)=1.1d_2(t)$ and $d_2(t)\le 5$, otherwise $d_2(t+1)=0.9d_2(t)$; the IP supplier selects S_s such that as its profit in t -th round is lower than that in (t -1)-th round, let $d_1(t+1)=0.9d_1(t)$. Fixed parameters with $x_{GB}(1)=0.1$, $y_0=0.9$, using the simulation method in (Ⅰ) and (Ⅱ), the result (Fig. 5) shows that about one third cases, GB and IP suppliers coexist; in most cases, with the GB supplier's price reaches a peak, the GB supplier earns a higher market share than the IP supplier. The range of average cost for a product is [1.15, 1.2]; social welfare declines compared to (Ⅰ) and (Ⅱ), which is not an obvious result.

(Ⅳ) The GB supplier (IP supplier) selects the strategy S_s (S_p) like the selection of the IP supplier (GB supplier) in (III). Using the initial parameters $x_{GB}(1)$, y_0 and simulation also like in (III), the result shows that, in 2/3 of cases, the GB supplier not only hold the price advantage, but also occupy the market.

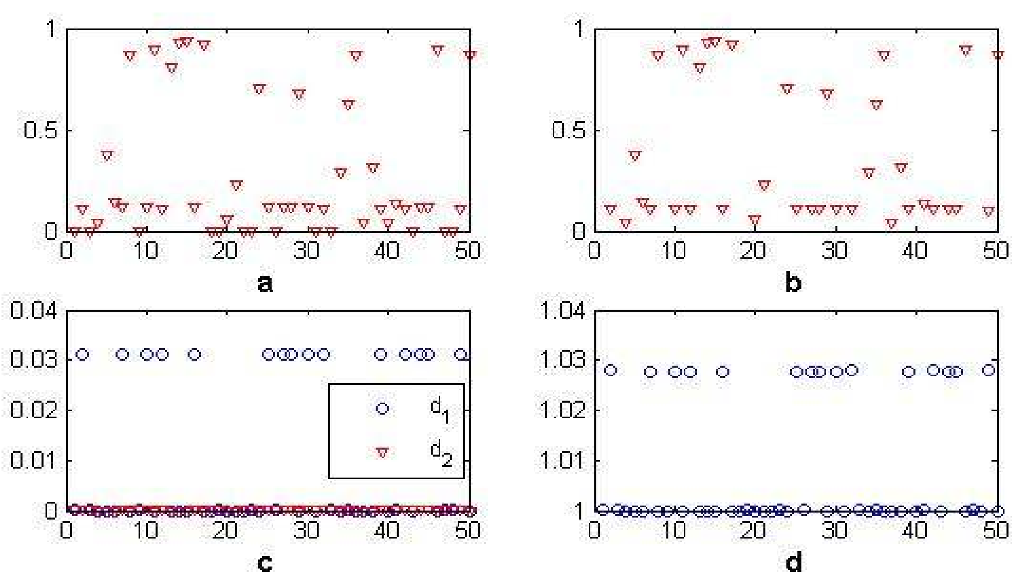

Fig. 3. a: The frequency of GB individuals; b: The percentage of the GB supplier' profit; c: The values of d_1 and d_2; d: Individuals' average cost for a unit product

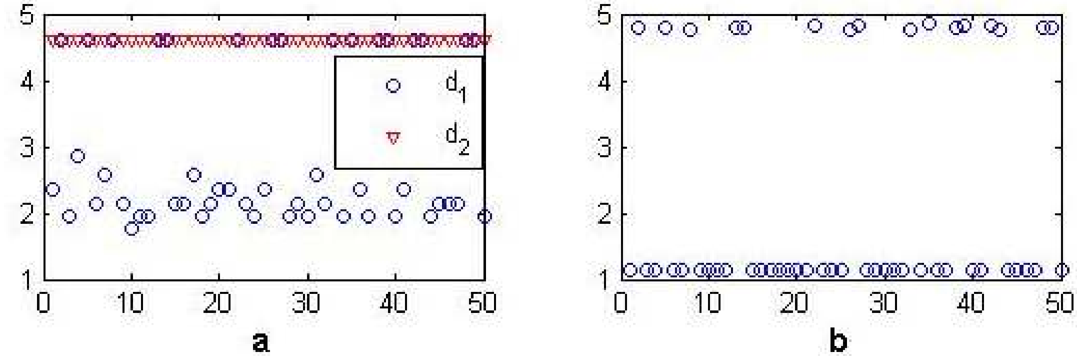

Fig. 4. a: The values of d_1, d_2; b: Individuals' average cost for a unit product

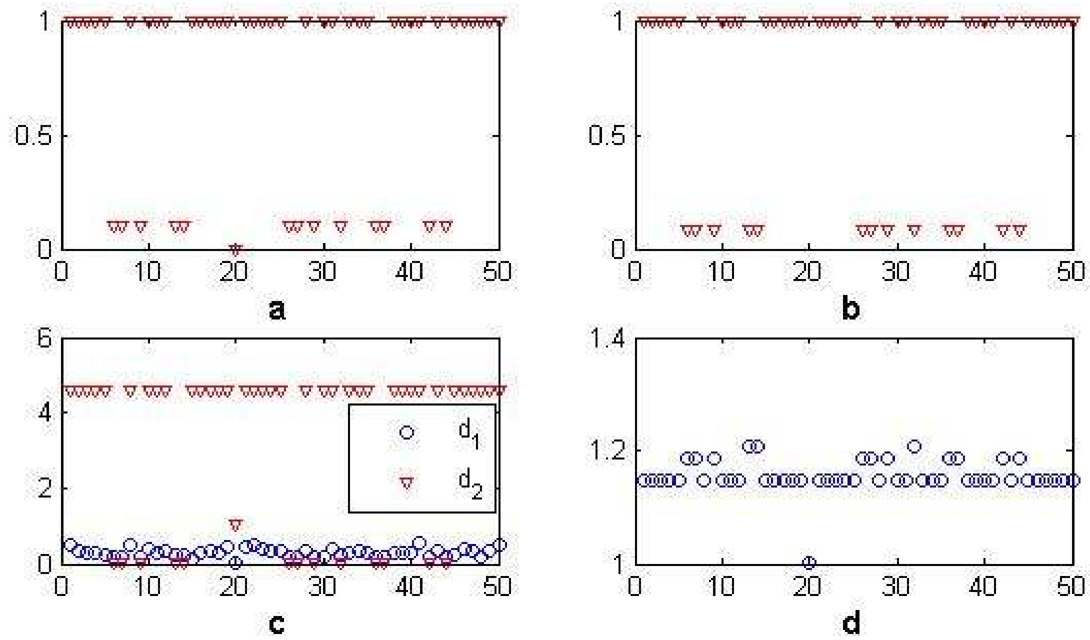

Fig. 5. a: The frequency of GB individuals; b: The percentage of the GB supplier' profit; c: The values of d_1 and d_2; d: Individuals' average cost for a unit product

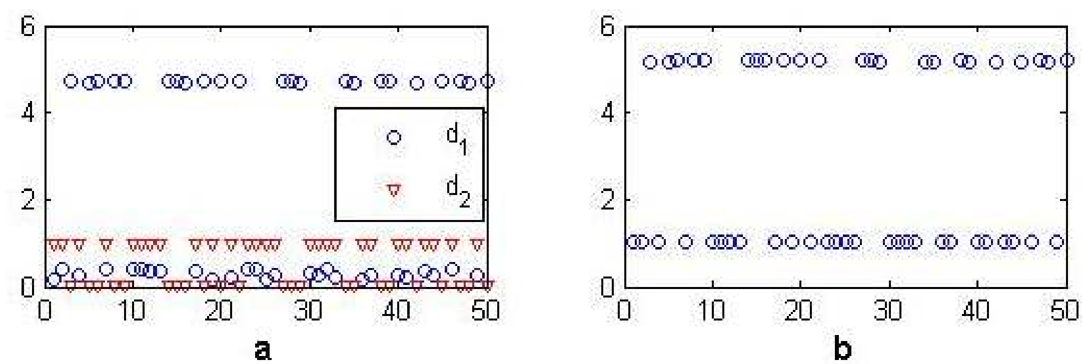

Fig. 6. a: The values of d_1, d_2; b: Individuals' average cost for a unit product

If we fix x_{GB} or y_0 in the simulations of (Ⅱ), (Ⅲ) and (Ⅳ), similar to the corresponding discussion in (I), as the other changes, we can also find the threshold which leads to the disappearance of IP individuals; when there are large gaps between GB and IP suppliers' discount, for example, $d_1(t)=0.9d_1(t-1)$, while $d_2(t)=0.5d_2(t-1)$, this will decrease the values of $d_1(t)$ and $d_2(t)$ in steady states.

4. RESULTS

Based on the above analysis, note that if there are no individuals choose IP in a population, that means the IP supplier will exit the market. Simulation results demonstrate that, from the individuals' view point, there are two factors which affect the coexistence of GB and IP suppliers: (1) the frequency of IP individuals will decrease in the long run if GB individuals' initial frequency $x_{GB}(1)$ increase. (2) with the decreasing of decisions on best response, coexisting cases of GB and IP individuals will increase, that is, bounded rationality for individuals has positive effects on the diversity of suppliers.

On the other hand, suppliers' strategy has important influence for their coexistence. The IP supplier's strategy depends highly on the GB supplier' selection. As the discussion in (I), If the IP supplier follows the GB supplier by selecting the same kind strategy, it will lead to their coexistence within a large range of parameters and share the market together, but IP supplier will exit the market in some range of parameters; and differentiation of discounts can decrease individuals' average cost and will reveal the base price, this improve the welfare of society. The results of (Ⅲ) and (Ⅳ) show that, if one of suppliers prefer market scale, and the other prefer selling price, the GB supplier will have more opportunities to win market share.

At last, it is important to point out that if suppliers can cooperation (collusion, or a supplier act as two types of suppliers), the results of (Ⅱ) show that their earnings will increase.

5. CONCLUSION

In this paper, we establish an evolutionary game model for group buying games. Each of suppliers has two strategies and each individual in a population also has two kinds of strategies. Based on the simulation, the diversity of suppliers may have positive contributions for social welfares under some of their strategy profiles; however, different types acted by only a supplier will decrease social welfares because buyers' bounded rationality.

Therefore, these evolutionary analysis results may contribute that, in a group buying market, how to design a mechanism to maintain the diversity of suppliers and to increase the social welfare.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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